Control Mechanism of Surface Subsidence and Overburden Movement in Backfilling Mining based on Laminated Plate Theory

Zhengzheng Cao¹, Feng Du²³⁴, Ping Xu¹, Haixiao Lin¹, Yi Xue³, Yuejin Zhou³

Abstract: The backfilling mining technology is a type of high-efficiency coal mining technology that is used to address the environmental issues caused by the caving mining technology. In this paper, the mechanical model of symmetrical laminated plate representing the overburden movement caused by the backfilling mining technology is established, and the governing differential equation of the motion of the overburden is derived. The boundary conditions of the mechanical model are put forward, and the analytical solution of the overburden movement and surface subsidence is obtained. The numerical model of the overburden movement and surface subsidence, under mining with backfilling, is established by means of the FLAC3D numerical software, which aims to systematically study the influence of backfilling compactness, mining thickness, and mining depth on the overburden movement and surface subsidence in backfilling mining. When the compactness \( \eta \) is less than 70\%, the overburden movement and surface subsidence is greater, while when \( \eta \) is greater than 70\%, the overburden movement and surface subsidence is reduced significantly. On this basis, the control mechanism of surface subsidence and overburden movement in backfilling mining is obtained. The suitable backfilling compactness is the key to controlling surface subsidence and overburden movement in backfilling mining.

Keywords: Symmetrical laminated plate theory, surface subsidence, overburden movement, backfilling mining.

1 Introduction

Due to the high demand of coal and the large spread of urbanization, mining is seeking ways to obtain coal that is deposited under buildings, railways and bodies of water. Mining under structures can create hazards, such as overburden movement and surface subsidence, along with the need to dispose of the waste material that is typically stacked on the mine (such as coal gangue and coal ash). Backfilling mining technology is a safe...
and highly efficiency coal mining technique that has been developed specifically for this purpose and has already proven to be an effective practice [Qian (2010); Zhou, Guo, Cao, and Zhang (2013); Cao, Zhou, Xu, and Li (2014); Xue, Gao, and Liu (2015); U. Babuscu (2017)]. According to the mining practice, the overburden rock in backfill mining method presents continuous bending deformation, while periodic breakage of overburden rock is mainly shown in the caving mining method [Zhang, Miao, and Guo (2009); Miao (2012); Ding, Zhou, Xu, Peng, and Cao (2013); Ju, Li, Zhang, Miao, and Liu (2014); Zhou, Tian, and Zhang (2016)]. Considering that the overburden rock behaves dramatically differently during backfill mining method than during the caving mining method, in-depth research needs to be conducted on the overburden movement and surface subsidence mechanisms under backfill mining and a corresponding strata control theory of backfill mining needs to be established [Pankow, Waas, and Yen (2012); Miao, Wei, and Yan (2013); Xie, Xu, Wang, Guo, and Liu (2014); Xin and Ji (2016)].

Currently, the research on strata control under backfill mining mainly focuses on the mechanical model of a rock beam, which describes the central region of the working face reasonably [Qian, Miao, and Xu (2008); Zhang, Li, and An (2010); Miao, Huang, and Ju (2012); Ning, Wang, Liu, Qian, and Sun (2014); Jia, Qiao, and Jiang (2016)]. As this rock beam model attribute the movement of overburden rock to the plane strain type, the deformation of the edge of the working face cannot be illustrated properly [Miao, Zhang, and Guo (2010); Zhou, Chen, Zhang, and He (2012); Cao and Zhou (2015); Yi and Cui (2015); Xue, Ranjith, Gao, Zhang, Cheng, Chong, and Hou (2017)]. Therefore, based on the deformation characteristics of overburden rock under backfill mining, the mechanical model of a symmetrical laminated plate is adopted in this paper, which is beneficial to an in-depth analysis of the deformation of the overburden strata under backfill mining.

2 Mechanical analysis of overburden movement under backfill mining

The fully mechanized mining with solid backfilling consists of five key production systems, namely the coal transport system, auxiliary transport system, ventilation system, solid waste transport system, and a backfill system. The coal transport system, auxiliary transport system, and ventilation system are the same as those used in traditional fully mechanized coal mining. The solid waste transport system transports the solid waste from the surface to the underground working face, and the backfill system places the dense backfill material in the gob area, which is developed to handle solid backfill materials as an integral part of the original fully mechanized coal mining system.

As continuous slight-bending deformation of overburden strata is produced under backfill mining, the overburden strata structure is simplified into a symmetrical laminated plate structure consisting of several isotropic strata based upon the hierarchical structure and soft-hard alternating characteristics of overburden strata.

When the working face is advanced under backfill mining, concurrent deformation of overburden strata occurs, the deflection expression in various strata become consistent, and combined with the deflection expression in plate theory [Xu (2005)]:

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\[ w = \frac{16q_0}{\pi^6 D} \sum_{m=1,3,5,...}^{\infty} \sum_{n=1,3,5,...}^{\infty} \frac{\sin(m \pi x / a) \sin(n \pi y / b)}{mn^2 / a^2 + n^2 / b^2} \]

The \( q_0 \) is the topsoil load, and the coefficients \((a, b)\) in various strata are consistent. And combined with the bending rigidity, the expression of curvature and torsion rate in plate theory [Xu (2005)]. Therefore:

\[
\begin{align*}
M_{xx}^i (1 - \mu_i^2) &= M_{xx}^2 (1 - \mu_2^2) = \ldots = M_{xx}^j (1 - \mu_j^2) = \ldots = M_{xx}^n (1 - \mu_n^2) \\
E_i h_i^3 (\chi_i^2 + \mu_i \chi_i^3) &= E_h^3 (\chi_i^2 + \mu_i \chi_i^3) = \ldots = E_i h_i^3 (\chi_i^2 + \mu_i \chi_i^3) = \ldots = E_h^3 (\chi_i^2 + \mu_i \chi_i^3)
\end{align*}
\]

\[
M_{yy}^i (1 - \mu_i^2) = M_{yy}^2 (1 - \mu_2^2) = \ldots = M_{yy}^j (1 - \mu_j^2) = \ldots = M_{yy}^n (1 - \mu_n^2) \\
E_i h_i^3 (\chi_i^2 + \mu_i \chi_i^3) = E_h^3 (\chi_i^2 + \mu_i \chi_i^3) = \ldots = E_i h_i^3 (\chi_i^2 + \mu_i \chi_i^3) = \ldots = E_h^3 (\chi_i^2 + \mu_i \chi_i^3)
\]

The overburden movement could be controlled effectively through the backfilling mining technology [Miao, Huang, and Ju (2012)]. Namely, the curvature rate \((\chi_x, \chi_y)\) in overburden strata could be regarded as the tiny value. The rate of curvature and torsion in various strata become consistent under backfill mining, therefore:

\[
\begin{align*}
M_{xx}^i (1 - \mu_i^2) &= M_{xx}^2 (1 - \mu_2^2) = \ldots = M_{xx}^j (1 - \mu_j^2) = \ldots = M_{xx}^n (1 - \mu_n^2) \\
E_i h_i^3 (\chi_i^2 + \mu_i \chi_i^3) &= E_h^3 (\chi_i^2 + \mu_i \chi_i^3) = \ldots = E_i h_i^3 (\chi_i^2 + \mu_i \chi_i^3) = \ldots = E_h^3 (\chi_i^2 + \mu_i \chi_i^3)
\end{align*}
\]

Because the total bending moment and torque of the laminated plate structure of overburden rock are shared by each section of single rock layer, and assuming that \( m_{xx}^i, m_{yy}^i, m_{xy}^i, m_{yx}^i \) as the moment and torque per unit length of the cross section:

\[
\begin{align*}
m_{xx}^i &= E_i h_i^3 \left( \frac{\sum_{k=1}^{n} (1 - \mu_k^2)}{(1 - \mu_i^2) \sum_{k=1}^{n} E_i h_i^3} \right) m_{xx}^{mn} \\
m_{yy}^i &= \frac{E_i h_i^3 \sum_{k=1}^{n} (1 - \mu_k^2)}{(1 - \mu_i^2) \sum_{k=1}^{n} E_i h_i^3} m_{yy}^{mn} \\
m_{xy}^i &= \frac{G_i h_i^3 \sum_{k=1}^{n} (1 + \mu_k)}{(1 + \mu_i) \sum_{k=1}^{n} G_i h_i^3} m_{xy}^{mn} \\
m_{yx}^i &= m_{xy}^i \\
\end{align*}
\]
Combined with the expression of the deflection $w_i$ per unit length of the $i$th stratum in cross section [Xu (2005)]:

$$m_x = -D \left( \frac{\partial^2 w_i}{\partial x^2} + \mu_i \frac{\partial^2 w_i}{\partial y^2} \right) \frac{(1-\mu_i^2) \sum_{k=1}^{n} E_i h_k^3}{E_i h_i \sum_{k=1}^{n} (1-\mu_k^2)}$$

$$m_y = -D \left( \frac{\partial^2 w_i}{\partial y^2} + \mu_i \frac{\partial^2 w_i}{\partial x^2} \right) \frac{(1-\mu_i^2) \sum_{k=1}^{n} E_i h_k^3}{E_i h_i \sum_{k=1}^{n} (1-\mu_k^2)}$$

$$m_{xy} = -D(1-\mu_i) \frac{(1+\mu_i) \sum_{k=1}^{n} G_i h_k^3}{G_i h_i \sum_{k=1}^{n} (1+\mu_k)}$$

Omitting the hexahedron $h \cdot dx \cdot dy$ from the symmetrical laminated plate mechanical model (Fig. 1), torque balance equations can be obtained with the torque shaft axis in parallel to the axis $y$ and $x$ through differential block center, respectively. Meanwhile, the force balance equation along the $z$ axis is as follows:

$$\sum (M_z) = -f_{za} \frac{dx}{2} + (f_{za} + \frac{\partial f_{za}}{\partial x} dx) \frac{dx}{2} + m_x \frac{dx}{2} + \frac{\partial m_x}{\partial x} dx - m_{xy} \frac{dy}{2} + \frac{\partial m_{xy}}{\partial y} dy = 0$$

$$\sum (M_y) = f_{ya} \frac{dy}{2} + (f_{ya} + \frac{\partial f_{ya}}{\partial y} dy) \frac{dy}{2} + m_y \frac{dy}{2} + \frac{\partial m_y}{\partial y} dy - m_{xy} \frac{dx}{2} + \frac{\partial m_{xy}}{\partial x} dx = 0$$

$$\sum F_z = -f_{za} dy + (f_{za} + \frac{\partial f_{za}}{\partial x} dx) dy - f_{ya} dx + (f_{ya} + \frac{\partial f_{ya}}{\partial y} dy) dx + q dx dy = 0$$

**Figure 1:** Mechanical model of the hexahedron
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Therefore:

\[ \frac{\partial^4 w_i}{\partial x^4} + 2 \mu_i + 2(1 - \mu_i^2) \frac{(1 + \mu_i) \sum_{k=1}^{n} G_k h_k^3}{(1 - \mu_i^2) \sum_{k=1}^{n} E_k h_k^3} \left[ \frac{\partial^4 w_i}{\partial x^2 \partial y^2} + \frac{\partial^4 w_i}{\partial y^4} \right] = q \frac{E_i h_i^3 \sum_{k=1}^{n} (1 - \mu_k^2)}{D_i (1 - \mu_i^2) \sum_{k=1}^{n} E_k h_k^3} \]  

(7)

If \( A_i = \mu_i + 2(1 - \mu_i^2) \frac{(1 + \mu_i) \sum_{k=1}^{n} G_k h_k^3}{(1 - \mu_i^2) \sum_{k=1}^{n} E_k h_k^3} \) and \( B_i = \frac{(E_i h_i^3) \sum_{k=1}^{n} (1 - \mu_k^2)}{D_i (1 - \mu_i^2) \sum_{k=1}^{n} E_k h_k^3} \), the governing differential equation of overburden movement is expressed as formula (8), based on the symmetrical laminated plate theory:

\[ \frac{\partial^4 w_i}{\partial x^4} + 2 A_i \frac{\partial^4 w_i}{\partial x^2 \partial y^2} + \frac{\partial^4 w_i}{\partial y^4} = qB_i \]  

(8)

The load \( q(x, y) \) is expressed as follows:

\[ q(x, y) = q_0 + p = q_0 - kw \]  

(9)

The distributed load \( q_0 \) is the topsoil load, and \( p \) is the support force by the backfill solid, which can be described by the wrinkle foundation assumption. Combining formula (8) with formula (9) gives:

\[ \frac{\partial^4 w_i}{\partial x^4} + 2 A_i \frac{\partial^4 w_i}{\partial x^2 \partial y^2} + \frac{\partial^4 w_i}{\partial y^4} + kB_i w_i = q_0 B_i \]  

(10)

The boundary condition is regarded as simply supported on four sides when the burial depth of a coal seam is relatively shallow. The deflection expression is obtained:

\[ w_i = \frac{16 B_0 q_0}{\pi^2} \sum_{m=1,3,5} \sum_{n=1,3,5} \frac{\sin(m \pi x / a) \sin(n \pi y / b)}{mn} \left[ \pi^4 \left( m^4 / a^4 + 2 A m^2 n^2 / a^2 b^2 + n^4 / b^4 \right) + kB_i \right] \]  

(11)

The deflection \( w_i \) of the \( i \) overburden strata is expressed in formula (11), and it can be seen that the deflection \( w_i \) is hinged on \( q_0 \) (topsoil load) and \( k \) (foundation coefficient). The distributed topsoil load \( q_0 \) is proportional to the mining depth \( H \), while the foundation coefficient \( k \) is related to the backfilling compactness \( \eta \) and the mining thickness \( h \). In order to further study the influence of backfilling compactness, mining thickness and mining depth on the movement of overburden rock, the numerical method with FLAC3D numerical software is introduced.
3 Numerical simulation of overburden strata under backfill mining

3.1 Numerical calculation model

FLAC3D (Fast Lagrangian Analysis of Continua) is the finite-difference computer codes for geomechanics applications, which can simulate a full range of nonlinear static and dynamic mechanical problems. Therefore, it is suited for modeling the deformation of the overburden strata and surface subsidence controlled Backfilling mining technology. This paper establishes the numerical calculation model of strata movement under filling mining using FLAC3D numerical software as the analysis platform.

The size of the numerically calculated mechanical model is 1500m × 500m × 400m, and the working face is advanced from 400 m to 1000 m with an advancing length of 600 m. The boundary conditions are set so that the bottom of the model restricts the vertical displacement and the four sides of the model restrict the horizontal displacement. The mechanical parameters of each single strata and the backfill solid are shown in Tab. 1 and Tab. 2.

Table 1: Mechanical parameters of each single strata

<table>
<thead>
<tr>
<th>No.</th>
<th>lithology</th>
<th>depth /m</th>
<th>Density /kg·m⁻³</th>
<th>bulk modulus /GPa</th>
<th>shear modulus /GPa</th>
<th>tensile strength /MPa</th>
<th>Cohesion /MPa</th>
<th>internal friction angle /°</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>topsoil</td>
<td>135</td>
<td>1300</td>
<td>5.6</td>
<td>1.85</td>
<td>1.0</td>
<td>1.5</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>sandstone</td>
<td>35</td>
<td>2500</td>
<td>22.7</td>
<td>11.70</td>
<td>5.0</td>
<td>8.3</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>mudstone</td>
<td>40</td>
<td>2400</td>
<td>8.3</td>
<td>3.80</td>
<td>1.5</td>
<td>6.1</td>
<td>30</td>
</tr>
<tr>
<td>4</td>
<td>sand and mudstone</td>
<td>65</td>
<td>2300</td>
<td>11.4</td>
<td>5.80</td>
<td>2.0</td>
<td>6.5</td>
<td>31</td>
</tr>
<tr>
<td>5</td>
<td>sandstone</td>
<td>36</td>
<td>2300</td>
<td>15.2</td>
<td>7.80</td>
<td>5.0</td>
<td>6.4</td>
<td>31</td>
</tr>
<tr>
<td>6</td>
<td>mudstone</td>
<td>20</td>
<td>2400</td>
<td>8.3</td>
<td>3.80</td>
<td>1.0</td>
<td>6.1</td>
<td>30</td>
</tr>
<tr>
<td>7</td>
<td>sandstone</td>
<td>36</td>
<td>2500</td>
<td>16.7</td>
<td>10.00</td>
<td>5.0</td>
<td>8.3</td>
<td>32</td>
</tr>
<tr>
<td>8</td>
<td>coal seam</td>
<td>3</td>
<td>1800</td>
<td>4.2</td>
<td>1.09</td>
<td>0.5</td>
<td>2.1</td>
<td>28</td>
</tr>
<tr>
<td>9</td>
<td>sandstone</td>
<td>30</td>
<td>2500</td>
<td>16.7</td>
<td>10.00</td>
<td>5.0</td>
<td>8.3</td>
<td>32</td>
</tr>
</tbody>
</table>

Table 2: Mechanical parameters of the backfill solid

<table>
<thead>
<tr>
<th>η/%</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>K/ GPa</td>
<td>0.009</td>
<td>0.013</td>
<td>0.022</td>
<td>0.057</td>
<td>0.179</td>
</tr>
<tr>
<td>G/GPa</td>
<td>0.006</td>
<td>0.007</td>
<td>0.013</td>
<td>0.022</td>
<td>0.030</td>
</tr>
</tbody>
</table>

3.2 Numerical calculation scheme

The backfilling compactness is the ratio of the mass of the backfill body backfilled into the gob to the mass of raw coal, which is regarded as a direct control indicator which
could guarantee the backfill effect. Therefore, it is important to design the backfilling compactness by considering the condition of surface structures in the mining area to guarantee that sufficient solid backfill material is used so as to control overburden movement and surface subsidence.

(1) The mining thickness is 3 m and the mining depth is 370 m. The backfilling compactness $\eta$ is set to 50%, 60%, 70%, 80% and 90% in order to research the effects of backfilling compactness on the overburden movement and surface subsidence.

(2) The backfilling compactness is 80% and the mining depth is 370 m. The mining thickness $h$ is set to 2 m, 2.5 m, 3 m, 3.5 m and 4 m in order to research the effects of mining thickness on the overburden movement and surface subsidence.

(3) The backfilling compactness is 80% and the mining thickness is 3 m. The mining depth $H$ is set to 300 m, 350 m, 400 m, 450 m and 500 m in order to research the effects of mining depth on the overburden movement and surface subsidence.

3.3 Influence of backfilling compactness on overburden movement and surface subsidence

The sinking curve of the key strata in different backfilling compactness is shown in Fig. 2. It can be clearly seen that when the working face advances, large sinking deformation still exists when $\eta$ is 50% and 60%, while the key strata only appear to have less continuous bending deformation when $\eta$ is larger than 70%. When $\eta$ increases from 50% to 70%, the amount of sinking that the key strata undergo reduces from 1500 mm to 690 mm, a total decrease of 810 mm.

![Figure 2: The sinking curve of the key strata in various backfilling compactness](image-url)

The sinking curve of surface subsidence with various values of $\eta$ is shown in Fig. 3. It can be observed that with an increase of $\eta$, the surface subsidence quantity decreases simultaneously. When $\eta$ is larger than 70%, the surface subsidence quantity reduces significantly. For instance, when $\eta$ increases from 50% to 70%, the surface subsidence
quantity reduces from 1400 mm to 670 mm, a total decrease of 730 mm.

Figure 3: The sinking curve of surface subsidence in various backfilling compactness

3.4 Influence of mining thickness on overburden movement and surface subsidence

The sinking curve of the key strata in various mining thickness is shown in Fig. 4. It can be clearly seen that an approximate linear relationship is observed between mining thickness and overburden movement. When $h$ increases from 2.0 m to 4.0 m, the amount of sinking that the key strata undergoes increases from 195 mm to 288 mm, a total increase of 93 mm.

Figure 4: The sinking curve of the key strata in various mining thickness
The sinking curve of surface subsidence with various values of \( h \) is shown in Fig. 5. It can be observed that a linear relationship also exists between mining thickness and surface subsidence quantity approximately. When \( h \) increases from 2.0 m to 4.0 m, the surface subsidence quantity increases from 175 mm to 260 mm, a total increase of 85 mm.

**Figure 5:** The sinking curve of surface subsidence in various mining thickness

3.5 Influence of mining depth on overburden movement and surface subsidence

The sinking curve of the key strata in various mining depth is shown in Fig. 6. It can be clearly seen that the overburden movement is proportional to mining depth approximately. When \( H \) increases from 300 m to 500 m, the amount of sinking that the key strata undergoes increases from 218 mm to 284 mm, a total increase of 66 mm.

**Figure 6:** The sinking curve of the key strata in various mining depth
The sinking curve of surface subsidence with various values of $H$ is shown in Fig. 7. It can be observed that the quantity of surface subsidence increases with the growth of $H$, and the increasing gradient of surface subsidence decreases with the increase of $H$. When $H$ increases from 300 m to 500 m, the surface subsidence quantity increases from 205 mm to 232 mm, a total increase of 27 mm.

![Figure 7: The sinking curve of surface subsidence in various mining depth](image)

4 Conclusions

(1) Based on the technological principle of fully mechanized mining with solid backfilling, the symmetrical laminated plate mechanical model of overburden movement and surface subsidence is established in backfilling mining technology; on this basis, the governing differential equation of the symmetrical laminated plate mechanical model is put forward, and the analytical displacement expression of overburden strata is obtained with the definite solution condition.

(2) Simulation results indicate that the influence of backfilling compactness ($\eta$) on the sinking amount of overburden strata and surface subsidence is more significant compared with that of mining thickness and mining depth. When $\eta$ is less than 70%, the sinking amount of overburden strata and surface subsidence is considerable; whereas when $\eta$ is greater than 70%, the sinking quantity of overburden strata and surface subsidence is significantly reduced.

(3) The control mechanism of surface subsidence and overburden movement in solid backfilling mining is obtained. The key to controlling surface subsidence is to restrict the deformation and fracture of key strata in overburden structure. In engineering practice, the suitable solid backfilling mining scheme could be chosen by reference to the change law of key strata with backfilling compactness, thus the surface subsidence and overburden deformation is confined within the specified range.
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