Numerical and Experimental Investigations of Jet Impingement on a Periodically Oscillating-Heated Flat Plate

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Abstract: In the present paper, the impingement of air jet on a heated flat plate subjected to a periodic oscillation is numerically and experimentally investigated. The motivation of the present research is the desire to enhance the heat transfer characteristics during the cooling process of a heated flat plate which can be found in many relevance industrial applications. In order to improve the heat transfer characteristics, a novel idea is utilized, where a periodical oscillation movement in form of sine wave produced from a Scotch yoke mechanism is applied to the heated flat plate. The obtained numerical results showed a good agreement with the performed experimental measurements. Moreover, an increase of the Nusselt number is obtained from an oscillated plate in comparison with that obtained from a fixed plate. This reveals the enhancement of the heat transfer characteristics of a heated flat plate as a result of the applied periodic oscillation movement.

Keywords: Numerical simulation, Experimental measurements, Jet impingement, heat transfer, periodically oscillating flat plate, turbulence modeling.

1 Introduction

Recently, the technology of jet impingement cooling of heated surfaces has received a great attention due its heat transfer effectiveness in many industrial and engineering applications, such as: cooling of photovoltaic cells [Royne and Dey (2004)], electronics cooling [Narumanchi, et al. (2003)], steel making process [Viskanta and Incropera (1990)], cooling of gas turbine blades [Liu and Feng (2011)], and even in food-processing operations [Sarkar, et al. (2006)]. Consequently, extensive researches have been carried out over several decades to explore the thermo-

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fluid dynamics and behavior of the impinging jets on fixed surfaces, see [Kubacki and Dick (2011)] and [Pakhomov and Terekhov (2011)].

Different parameters can affect the overall efficiency of the jet impingement cooling process on fixed flat plate, e.g. impinging jet momentum, nozzle distance from the impingement surface and the turbulence characteristics of the jet, see for more details [Balabel (2012); Sharif and Mothe (2010)]. Recently, a great attention has been given to the prediction of the Reynolds stresses of the impinging jet on fixed plate as they usually dominate the process of heat and mass transfer [Balabel and El-Askary (2011)]

Recently, many experimental measurements have been performed in order to predict the turbulence as well as heat transfer characteristics for impinging jet on a fixed flat plate, see for more details [Choo et al. (2009)]. However, such experimental investigations are normally carried out for illustrating the fluid dynamics and for validating the numerical simulations for such problems. Moreover, such experimental measurements have fascinated several problems due to the impinging process especially the measurements in the attached thermal boundary layer to the impinging surface and near of the impinging point, see [Zhou et al. (2009)]

More recently, the carefully executed simulations can virtually replace experiments. The numerical simulation has become a standard tool in many areas of thermo-fluid dynamics where accurate predictions of velocity and vorticity fields are not available experimentally. The superior variability of numerical simulations over the experimental measurements is clearly shown in the choice of initial and the boundary conditions as well as the different fluid properties. This makes the numerical simulation an important procedure in predicting the important characteristics of impinging jets. However, many numerical methods challenges are existed, such as the numerical instability, turbulence modeling and the choice of the computational grids, see [Xu and Hangan (2008)]

As an important feature of the impinging jet process, the high heat transfers rates which occur in the impingement region of the jets decays rapidly away from the stagnation point. Consequently, for large impinging surfaces or a desired rapid heat transfer process, it is strongly recommended to use multiple impinging jets. Therefore, extensive researches on the multiple jets have been carried out in order to determine the effect of impinging jets arrays on the heat transfer characteristics of the impinging surface, see [Whelan and Robinson (2009)]. However, such investigations, that considered jet array impingement on a stationary surface, showed that cross flow and adjacent jet interactions decrease the magnitude of heat transfer coefficient of individual jets.

In order to enhance the heat transfer characteristics for impinging jet over a heated
flat plate, a specified movement for the plate is imposed. This is considered as an alternative to the impinging jet arrays. A number of investigations have concerned with the impinging of confined jet arrays on a moving surface with a specified horizontal velocity, e.g. [Aldabbagh and Mohamad (2009)]. The main objective of such researches was to find out the role of the surface motion in the development of the impinging turbulent slot jets. In general, it is concluded that remarkable enhancement of heat transfer rate was obtained by larger moving block velocity. However, the resulting flow field from the confined turbulent jet arrays is significantly affected. Moreover, the resulting acceleration and deceleration motions of the moving bar during the initial and final stages of plate movement are unavoidable due to direct contact between the moving block and the heat plate; see [Fu et al. (2007)]

In the present paper, the impinging jet cooling process for a heated flat plate is carried out numerically and experimentally. The flat plate is allowed to oscillate according to sinusoidal motion resulting from a Scotch yoke mechanism and fitted with the flat plate. This may avoid the acceleration and deceleration motions of the horizontally moving plate. The obtained numerical results for heat transfer coefficient are compared firstly with the experimental measurements for the purpose of validation. The effect of the periodic motion of the flat plate on the heat transfer characteristics is also discussed.

2 Physical and Mathematical formulation

The mathematical formulation of the impinging jet problem, shown in Fig. 1, can be summarized as a set of the governing conservation equations for mass, momentum and energy equations. As the issuing jet has a relatively high velocity, then the turbulent flow formulations should be considered. Consequently, the system of the governing equations consists of the Reynolds-Averaged form of the mass, momentum and energy equations. More details will be given in the following sections.

2.1 Reynolds-Averaged Navier-Stokes Equations

The Reynolds form of the continuity and momentum equations for turbulent flow, called here RANS equations, at each point of the flow field can be represented by the following equations after neglecting the body force:

\[
\frac{\partial}{\partial x_i} (r \rho U_i) = 0 \tag{1}
\]

\[
\frac{\partial}{\partial t} (r \rho U_i) + U_j \frac{\partial}{\partial x_j} (r \rho U_i) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial U_i}{\partial x_j} - \rho u'_i u'_j \right) \tag{2}
\]
\[
\frac{\partial}{\partial t} \left( \rho T \right) + U_j \frac{\partial}{\partial x_j} (r \rho T) = \frac{\partial}{\partial x_j} r \left( \frac{\mu}{Pr} \frac{\partial T}{\partial x_j} - \rho T' u'_j \right) \tag{3}
\]

where \( P, T \) and \( U_i \) are the mean pressure, temperature and velocity components, respectively. \( T' \) and \( u' \) are the fluctuating temperature and velocity components, respectively.

Several investigations have been published evaluating the performance of the different turbulence models that can be applied for predicting the turbulent characteristics in case of impinging jet on a fixed plate [Balabel and El-Askary (2011)]. Moreover, the evaluation of the heat transfer characteristics near the stagnation region has been proven to be difficult in case of using different kinds of turbulence models, see [Hosseinalipour and Mujumdar (1995)].

Here, the turbulent Reynolds stresses \(-\rho \overline{u'_i u'_j}\) are calculated by an appropriate turbulence model for closure.

\[
\mathcal{R}_{ij} = -\rho \overline{u'_i u'_j} = -\frac{2}{3} \rho k \delta_{ij} + 2 \mu_t S_{ij} \tag{4}
\]

where \( \delta_{ij} \) is the Kronecker delta and \( \overline{u'_i u'_j} \) are the average of the velocity fluctuations. The turbulent viscosity is defined as:

\[
\mu_t = \rho C_{\mu} k^2 / \varepsilon \tag{5}
\]

The turbulent heat flux \(-\rho \overline{T'u'_j}\) is obtained according to the simple gradient diffusion hypothesis (SGDH), see [Ince and Launder (1989)]:

\[
-\rho \overline{T'u'_j} = \frac{\mu_t}{\sigma_t} \left( \frac{\partial T}{\partial x_j} \right) \tag{6}
\]
2.2 Standard $k$-$\varepsilon$ Turbulence Model and $v^2 - f$ model

In order to calculate the turbulent quantities, the turbulent kinetic energy $k$ and its dissipation rate $\varepsilon$ should be estimated by solving the following equations:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = \frac{\partial}{\partial x_j} \left[ (\mu + \frac{\mu_t}{\sigma_k}) \frac{\partial k}{\partial x_j} \right] + \rho (Pr - \varepsilon)$$  \hspace{1cm} (7)

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_j}(\rho u_j \varepsilon) = \frac{\partial}{\partial x_j} \left[ (\mu + \frac{\mu_t}{\sigma_\varepsilon}) \frac{\partial \varepsilon}{\partial x_j} \right] + \rho \left( \frac{\rho}{Time} (C_1 \varepsilon Pr - C_2 \varepsilon \varepsilon) \right)$$  \hspace{1cm} (8)

where $Pr = P_k/\rho$ and $\varepsilon$ represents dissipation rate of the turbulent kinetic energy, $k$, respectively; while $\sigma_k$ and $\sigma_\varepsilon$ are model constants. The production rate is related to the mean strain of the velocity field through the Boussinesq assumption. That is,

$$Pr = \mu_t S^2$$  \hspace{1cm} (9)

where $S$ is defined as:

$$S = \sqrt{\frac{1}{\rho} \left( \frac{\partial u_i}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) \frac{\partial u_i}{\partial x_j}}$$  \hspace{1cm} (10)

The $v^2$ transport equation is:

$$\frac{\partial}{\partial t}(\rho v^2) + \frac{\partial}{\partial x_j}(\rho u_j v^2) = \frac{\partial}{\partial x_j} \left[ (\mu + \frac{\mu_t}{\sigma_k}) \frac{\partial v^2}{\partial x_j} \right] + \rho (kf - 6v^2 \varepsilon / k)$$  \hspace{1cm} (11)

While the elliptic-relaxation equation $f$ can be represented as:

$$L^2 \frac{\partial^2 f}{\partial x_j^2} - f = \frac{1}{T} \left[ \left( C_1 - 6 \right) \frac{v^2}{k} - \frac{2}{3} (C_1 - 1) \right] - C_2 \frac{Pr}{k}$$  \hspace{1cm} (12)

and the turbulent length scale $L$ is determined from the values of $k$ and $\varepsilon$ as follows:

$$L = C_L \max \left[ \frac{k^{3/2}}{\varepsilon}, C_\eta \frac{(\mu/\rho)^{3/4}}{\varepsilon^{1/4}} \right]$$  \hspace{1cm} (13)

The various model constants are given in the following table, [Lauder and Spalding (1974)]:

$\sigma_k, \sigma_\varepsilon, C_\tau, C_{\varepsilon1}, C_{\varepsilon2}, C_{\mu}$ are given as 1, 1.3, 0.85, 1.44, 1.92, and 0.09 respectively. The model constant in case of using $v^2$-$f$ can be given as [Balabel and El-Askary (2011)]:

$C_\mu = 0.22; \quad \sigma_k = 1; \quad \sigma_\varepsilon = 1.3; \quad C_{\varepsilon1} = 1.4(1 + 0.05 \sqrt{k/v^2}); \quad C_{2\varepsilon} = 1.9;
C_1 = 1.4; \quad C_2 = 0.3; \quad C_L = 0.23; \quad C_\eta = 70.$
3 Numerical Procedure

The numerical method applied here for solving the set of the governing equations has been approved its originality and effectiveness in many previous papers of the present author [Balabel, (2011), (2011), (2012)]. Following the numerical scheme is briefly described.

3.1 Numerical Scheme

The present algorithm is based on the implicit fractional step-non iterative method to obtain the velocity and pressure filed in the computational domain. Assuming that the velocity field reaches its final value in two stages; that means:

\[ \mathbf{U}^{n+1} = \mathbf{U}^* + \mathbf{U}_c \]  \hspace{1cm} (14)

Whereby, \( \mathbf{U}^* \) is an imperfect velocity field based on a guessed pressure field, and \( \mathbf{U}_c \) is the corresponding velocity correction. Firstly, the ‘starred’ velocity will result from the solution of the momentum equations. The second stage is the solution of Poisson equation for the pressure:

\[ \nabla^2 p_c = \frac{\rho \alpha}{\Delta t} \nabla \cdot \mathbf{U}^* \]  \hspace{1cm} (15)

where \( \Delta t \) is the prescribed time step and \( p_c \) is called the pressure correction. Once this equation is solved, one gets the appropriate pressure correction and, consequently, the velocity correction is obtained according to the following equation:

\[ \mathbf{U}_c = -\frac{\Delta t}{\rho \alpha} \nabla p_c \]  \hspace{1cm} (16)

This fractional step method described above ensures the proper velocity-pressure coupling for incompressible flow field. However, the accurate solution of the surface pressure occurring at transient fluid interfaces of arbitrary and time dependent topology enables an accurate modeling of two- and three dimensional fluids.

3.2 Boundary Conditions

As known, at the pipe exit, the flow should be fully developed. In the computational work, it is suggested that an initial calculation should be done to generate fully-developed pipe flow profiles at the appropriate Reynolds number, which can then be used as inlet conditions for the impinging jet computation. The outlet plane should be placed at a sufficiently large radial distance so that errors arising from the application of zero-gradient (or similar) conditions will not significantly affect the region of interest. For the present measurements (extending to around \( r/D_j=8 \))
and the measurements in the literature used to verify the present model (extending to $r/D_j=8.8$), it is suggested that the outer radial boundary should be at $r/D_j=10$ or greater.

The boundary opposite the impingement wall is a surface across which fluid is entrained. One common method of dealing with such boundaries (in pressure-correction based finite-volume solvers) is to impose ambient pressure values at the boundary, and to allow fluid to be entrained at the rate necessary to satisfy continuity in the boundary cells. The computational domain is shown in Fig. (1).

4 Experimental Setup & Techniques

In the present work an experimental technique is used to address the physical parameters affecting the heat transfer associating with jet cooling process for a preheated surface. The test rig is designed and manufactured to facilitate the control and measurements of the preheated stainless steel sheet temperature during the jet cooling process. The plate is heated by a 2 kW electrical heater and equipped with temperature regulator. The surface exposed directly to normal air jet applied at a certain height as shown in Fig. 2.

![Figure 2: Layout of the experimental test rig](image-url)
The design of the test rig enables the heated sheet to be fixed or reciprocating during the experiments. Therefore a Scotch yoke mechanism is used to obtain simple harmonic motion. Thermocouples with type J is employed to measure the air and sheet temperature. The thermocouples are arranged in two arrays as shown.
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in Fig. 3.

It is dedicated firstly to measure the sheet temperature distribution; therefore they are embedded 1 mm below the upper surface as shown in Fig. 3. While the other array, penetrates the sheet which located at 1 mm above the surface to measure the air temperature in the conductive layer. The flow rate of the air jet is controlled via throttling valve and measured by using U tube manometer. At the beginning of each experimental cycle, the jet mass flow is adjusted to the desired flow rate and the heater is then fired to elevate the stainless steel sheet temperature up to certain value. Temperature is then maintained at the presetting value via temperature controller. The heat source is then shut down and the temperature distribution of both sheet and air is then recorded temporarily. The coolant temperature is also recorded by using the same thermocouple type and it is usually constant during the experimental cycle. The measured temperatures are then used to assign the local Nusselt number at each time step.

Nusselt number

In order to assign the Nusselt number, it is assumed that the heat is transferred initially from the hot plate surface by conduction through a very thin layer adjacent to the wall. The heat absorbed by this layer is then convected by the impinging air jet stream (ignoring the radiation mode). At thermal equilibrium condition, both amounts of heat transfer modes are equal, and then the Nusselt can be obtained as follows

\[
Nu = \frac{L \frac{dT}{dy}}{T_w - T_c} = \frac{L(T_w - T_a)}{(T_w - T_c)}
\]  

Therefore, the temperature gradient through the conductive layer is recorded via the previously mentioned two thermocouple arrays.

4.1 Uncertainty Analysis of Nusselt number

The uncertainty in Nusselt can arise from the error in temperature measurements. On the other hand, the wall temperature, conductive air layer and coolant temperatures were measured by the same thermocouple type, so the expected errors in their readings are the same. According to the measuring device the error in temperature reading is about \( \pm 0.1^\circ C \). Generally, the relative uncertainty is represented mathematically as follows.

\[
\frac{dNu}{Nu} = \frac{1}{Nu} \frac{\partial N u}{\partial T_w} \Delta T_w + \frac{1}{Nu} \frac{\partial N u}{\partial T_a} \Delta T_a + \frac{1}{Nu} \frac{\partial N u}{\partial T_c} \Delta T_c
\]  

(18)
Where, $\Delta T_w = \Delta T_a = \Delta T_c = \pm 0.1^\circ C$, finally, the relative uncertainty becomes as follows.

$$\frac{\Delta Nu}{Nu} = \frac{3}{(T_w - T_c)} \Delta T_w$$

(19)

Figure 4: Relative uncertainty in Nusselt (a) and Reynolds (b) numbers

The derivation of uncertainty in Nusselt number shows that only the wall and coolant temperature are effective as indicated above. While the coolant temperature is constant, the wall temperature becomes the dominant factor affecting the uncertainty. Figure 4.a shows the change in total relative uncertainty in $Nu$ with different wall temperature and at constant coolant temperature ($T_c = 30.4^\circ C$). As indicated in the figure, except at low wall temperature ($T_w = 40^\circ C$), the relative uncertainty lies below 1% threshold when $T_w = 60^\circ C$ or above.

4.2 Uncertainty Analysis of Reynolds number

In general, the change in Reynolds number associates the change in mass flow rate. The mass flow rate in turn is measured by standard orifice meter; therefore the U tube manometer reading $z$ is the governing factor in calculating mass flow rate and Reynolds number. Accordingly the relative uncertainty in both mass flow rate and Reynolds number is the same and it can be represented mathematically as follows:

$$\frac{dRe}{Re} = \frac{1}{Re} \frac{\partial Re}{\partial z} \Delta z$$

(20)

When performing derivative of $Re$ with respect to $z$, the following relative uncertainty in $Re$ becomes as follows.

$$\frac{\Delta Re}{Re} = \frac{1}{2z} \Delta z$$

(21)
where, \( z \) is the U tube manometer reading in mm and \( \Delta z \) is the expected error in measuring \( z \) and equal to 0.5 mm. Figure 4.b indicated how the relative uncertainty in \( Re \) changes with the entire operating range of U tube manometer reading. The figure exhibits a very low relative uncertainty even at lower range of \( z \). This is obviously due to the lower expected error in \( \Delta z \) compared to the measured values.

### 4.3 Kinematics of the Flat Plate

A Scotch-yoke mechanism is employed to drive the plate in a reciprocating motion. The advantage of using the scotch yoke instead of the crank slider mechanism confined in its linear simple harmonic motion at which the instantaneous velocity is symmetrical at the middle of the stroke, see Fig. 5.

![Figure 5: Kinematics of Scotch yoke mechanism (Rotational speed is 1500 rpm)](image)

The instantaneous plate displacement, velocity and its acceleration is calculated as follows:

**Displacement:**

\[
x = R \times [1 - \cos(\theta)]
\]

(22)

**Velocity**

\[
v = R \times \omega \times \sin(\theta)
\]

(23)

**Acceleration**

\[
a = \frac{dv}{dt} = R \times \omega^2 \times \cos(\theta)
\]

(24)
5 Verification

To verify the present computations we will first compare the Nusselt number distribution over fixed flat plate. The available experiments in the literature and used here for the verification of the computations provide an extensive set of measurements of a turbulent jet impinging orthogonally onto a large fixed plane surface. The Reynolds number considered is $2.3 \times 10^4$, while the attention is focused here for the height of the jet discharge above the plate of two diameters only, because of its generation of complex thermal-turbulence structure, see [Baugn et al., (1991)]. Computational results reported here will be focused on the distribution of Nusselt number on the impinging wall.

Figure 6: Distribution of the local Nusselt number on the impinging fixed plate

Figure 6 shows the good confrontation between $v^2 - f$ (compared with the standard $k - \varepsilon$ model) simulated in view of experimental literature Nusselt number of [Baugn et al. (1991)] at the stagnation point. Also for the prediction of the secondary maximum position (around $r/D = 2$ in the experiments, while at $r/D=1.8$ in the $v^2 - f$ model and not predicted in the standard $k - \varepsilon$ model). The Nusselt number decrease, obtained using $v^2 - f$ model for $r/D > 2$, is in good accordance
with the experimental data. Thus, this model has been chosen for the following numerical impingement jet calculations.

6 Results and Discussion

Computations are performed here to verify the present experimental measurements for the cases of the fixed plate as well as the oscillating plate. The distributions of the Nusselt number along the fixed as well as the oscillating wall (with frequency \( \omega = 10.476 \text{ rad/sec} \)) are shown in Fig. (2).

![Nusselt Number Distribution](image)

Figure 7: Distribution of the local Nusselt number on the impinging oscillating plate

It can be shown that, the predictions of the Nusselt number over either fixed or oscillating flat plate are in a fair agreement with the experimental measurements. The disappearance of the second peak of the Nusselt number here (fixed plate) is due to the lower Reynolds number of the flow issuing from the jet. In the present experiment, Reynolds number is 3200 compared with case of [Baughn et al., (1991)] in which Re=\(2.3 \times 10^4\). On the other hand the presence of oscillation cause a strong increase of Nusselt number at the central impinging point accompanied
with a strong near-wall Nusselt-number gradient. A noticeable deviation between the experimental data and the computation at the impinging point is clearly visible. This may be due the uncertainty of the experiments.

Figure 8: Computation of the temperature profiles at the symmetrical line at different time steps

It was difficult to measure the temperature profiles along the central line of the jet with successive time steps, because of the small space between the jet and the impinging wall. The computational results are only shown here to indicate the effects of oscillation the heat transfer behavior in such complex narrow space. From the preliminary computations, one notices that with the progressing the time near-wall temperature gradient decreases, because of the transferred heat from the wall in the direction of the jet source and hence the air between the jet and the impinging wall will be progressively heated. This will cause a strong cooling of the impinging wall compared with the fixed plate (time =1 sec), see Fig. (3).
7 Conclusion

In the present work, preliminary results from both numerical and experimental data are obtained for the Nusselt number distributions over a fixed flat plate as well as an oscillating wall with a defined oscillation speed. The comparison showed that an increase of the Nusselt number in case of oscillating flat plate is attained. This reveals the effect of plate oscillation on the enhancement of the heat transfer characteristics.

Different oscillation frequencies are planned to be considered in the future (experimental and numerical) studies by the present authors.

References


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