Mechanical Response Analysis and Safety Assessment of Shallow-Buried Pipeline under the Influence of Mining

Zhengzheng Cao¹, Yuejin Zhou¹,², Ping Xu¹ and Jiawei Li¹

Abstract: In accordance with the influence of underground mining on the deformation and failure of a shallow-buried gas pipeline, the pipe-soil interaction during mining is classified into two stages, namely coordinated deformation stage and partial hanging stage. According to the mechanical characteristics of the buried pipeline in each stage, the models of a) a beam on an elastic foundation, b) an elastic beam under uniform load, and c) a vertical and horizontal bending beam are introduced in a mining subsidence zone to mechanically analyze, respectively a) the pipeline in non-mining subsidence zone, b) the pipeline at the coordinated deformation stage, and c) the hanging stage. Mechanical model of segmented elastic beam of shallow-buried pipeline is established for each stage. Combined with the boundary conditions of each segmented elastic beam, the limiting condition of the pipe-soil interaction under the two stages is analyzed, and thus, the critical criterion of shallow-buried pipeline failure is obtained. Consequently, the mechanical analysis method on the deformation and failure of the shallow-buried pipeline is obtained. This method is used to analyze the practice of coal mining under a pipeline in Shanxi province, China. The analysis shows that the pipeline will likely fail in the pipe-soil partial hanging stage. Therefore, the pipe-soil interaction should be controlled within the coordinated deformation stage, in which the safe maximum length of working face should be limited to 435 m.

Keywords: Shallow-buried pipeline, mining subsidence, coordinated deformation stage, partial hanging stage, segmented elastic beam.

1 Introduction

At present, along with the large-scale exploitation of coal resources, mining subsidence zone is evolving and expanding dramatically, thus, posing serious threat

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to the shallow-buried gas pipelines [Mandolini, Minutolo, and Ruocco (2001); Sapountzakis and Mokos (2009); Sun and Su (2013)]. Hucka [Hucka, Blair, and Kimball (1986)] evaluated the stress and strain in such a pipeline according to the surface subsidence and bending deformation. In view of the long wall mining, Peng [Peng and Luo (1988)] proposed an analytical method of calculation for pipeline stress and strain. However, the study was valid only for the condition that the deformation of the buried pipeline should match with the soil surface deformation. Based on the surface subsidence and bending deformation, Iimura [Iimura (2004)] evaluated the stress level of a buried pipeline in a subsidence zone by combining the beam on elastic foundation method with the finite element method. Based on the probability integral method, Wang Xiaolin [Wang, Shuai, and Zhang (2011)] predicted the three-dimensional deformation of the earth’s surface in a mining subsidence zone, and established a mechanical model and calculation methods for buried pipeline at any position in the subsidence zone.

It can be seen that the state-of-the-art research mainly focuses on the deformation and displacement of buried pipeline only in its final state [Zhu, Chen, and Jiang (2008); Tuck, Lee, and Davidson (2013)]. However, as mining subsidence is a dynamic process, the deformation law of subsidence trough under stable conditions cannot represent the law of mining subsidence influence on the pipeline during the entire mining process. Consequently, the deformation and failure of buried pipeline may result during the mining subsidence [Chen, Liu, and Su (2011); Zhou, Guo, Cao, and Zhang (2013); Liu and Wang (2013)].

In order to assess the safety of buried pipeline in mining process during different stages, the corresponding mechanical model reflecting buried pipeline under the influence of mining should be established. Moreover, the failure criteria of buried pipeline under the limiting condition should be analyzed according to the different stages of pipe-soil interaction.

2 The pipe-soil interaction under the influence of mining

Mining practice shows that if the ratio of mining depth to thickness is greater than 30, a ground subsidence trough is formed as the working face of mining advances gradually [Tadeu, Pereira, and Godinho (2001); Zhang, Wang, and Ai (2010); Wang, Jiang, Zhu, Shan, and Wang (2013)], as shown in Figure 1.

In the dynamic evolution process of the subsidence trough, the soil surrounding a buried pipeline will move and deform, thereby causing the synchronous deformation of buried pipeline, which could be regarded as the coordinated deformation, as shown in Figure 2.

Along with the advancement of mining, the subsidence trough continues to grow
Figure 1: The ground subsidence trough.

Figure 2: The shallow-buried pipeline in coordinated deformation stage.

and expand. When the deformation of the surrounding soil exceeds the maximum allowable deformation of a buried pipeline, the settlement of the pipeline and the subsidence of the surrounding soil will no longer be synchronized; separation would gradually occur between the buried pipeline at the center of the subsidence trough and the soil below, resulting in the partial hanging stage of pipeline, as shown in Figure 3.
3 Mechanical analysis of a buried pipeline in the coordinated deformation stage

3.1 Mechanical model

3.1.1 Mechanical analysis of a buried pipeline in a non-subsidence zone

The coordinated deformation stage is shown in Figure 2. A buried pipeline in a non-subsidence zone is in a small deformation state, satisfying the basic condition of the model of a beam on an elastic foundation. Therefore, the model of semi-infinite continuous beam on elastic foundation is introduced in this study to conduct the mechanical analysis. In a non-subsidence zone, the overlying load above a pipeline and the bearing reaction below the pipeline are consistently in equilibrium, since the buried pipeline and surrounding soil are in static equilibrium over a long period of time. Thus, the control differential equation for the buried pipeline deformation in a non-subsidence zone is shown as follows:

$$EI \frac{d^4 y}{dx^4} + ky = 0 \quad (-\infty < x \leq 0) \quad (1)$$

where $EI$ represents the bending rigidity of the pipeline, $y$ represents the bending deflection, $k$ represents the coefficient of elastic foundation. When combined with the boundary conditions $y_{x=0} = 0$ and $y_{x=-\infty} = 0$, the solution is shown as follows:

$$y = \frac{M_B}{2EI\lambda^2} e^{\lambda x} \sin \lambda x \quad (2)$$
where \( M_B \) represents the bending moment at point \( B(x=0) \), and \( \lambda = (k/4EI)^{1/4} \). The deflection, slope and shear force at point \( B \) are shown as follows:

\[
\begin{aligned}
y_B &= y(x)_{x=0} = 0 \\
\theta_B &= y'(x)_{x=0} = \frac{M_B}{2EI\lambda} \\
Q_B &= -EIy'''(x)_{x=0} = -\lambda M_B \\
\end{aligned}
\]

(3)

With \( D \) as the external diameter, the bending strain of pipeline could be obtained from the curvature of each point as:

\[
\varepsilon(x) = \frac{Dy''(x)}{2} = \frac{DM_B}{2EI} e^{\lambda x} \cos(\lambda x) \quad (-\infty < x \leq 0)
\]

(4)

3.1.2 Mechanical analysis of a buried pipeline in a subsidence zone

Assuming that the length of the cross-section of a subsidence trough is \( 2l_0 \), the overlying soil load above a buried pipeline can be regarded as uniformly distributed in the pipe-soil coordinated deformation state. Thus pipeline \( BA \) could be seen as a mechanical model of the elastic beam under uniform load (figure 2). As the lateral load is the fourth-order derivative of deflection, the equation of beam deflection can be expressed as a fourth order polynomial:

\[
y = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0 \quad (0 < x \leq l_0)
\]

(5)

The deflection, rotation, bending moment and shear force of the pipeline at point \( B \) (\( x = 0 \)) are continuous, thus:

\[
\begin{aligned}
y_{x=0} &= 0, \quad y'_{x=0} = \frac{M_B}{2EI\lambda}, \quad y''_{x=0} = \frac{M_B}{EI}, \quad y'''_{x=0} = \frac{\lambda M_B}{EI} \\
\end{aligned}
\]

(6)

The slope of the pipeline at point \( A \) \( (x = l_0) \) is zero. The deflection of the pipeline at point \( A \) \( (x = l_0) \) could be regarded as the maximum surface subsidence \( y_{\text{max}} \), which could be calculated with the probability integral method. The coordinate system in Figure 1 should be transformed to Figure 2 as:

\[
y'_{x=l_0} = 0, \quad y_{x=l_0} = y_{\text{max}} = W(l_0 + r) - W(l_0 + r - l)
\]

(7)

Combining equation (6) with equation (7), the expression of \( M_B, a_0, a_1, a_2, a_3 \) and \( a_4 \) could be calculated as follows:

\[
\begin{aligned}
M_B &= \frac{W(l_0 + r) - W(l_0 + r - l)}{2\pi^2(4\lambda^2 + \frac{1}{4} + \frac{1}{\lambda})} \frac{\lambda l^2 + \frac{1}{4} + \frac{1}{\lambda}}{\lambda^2} \\
a_0 &= 0, \quad a_1 = -\frac{M_B}{2EI\lambda}, \quad a_2 = -\frac{M_B}{2EI} \\
a_3 &= -\frac{\lambda M_B}{6EI}, \quad a_4 = -\frac{M_B}{8EI\lambda^3}(\lambda^2 + 2l + \frac{1}{\lambda})
\end{aligned}
\]

(8)
The expression of deflection $y$ is as follows:

$$y = \frac{M_B}{2EI} \left( \frac{\lambda}{4l} + \frac{1}{2l^2} + \frac{1}{4\lambda l^3} \right) x^4 + \frac{\lambda}{3} x^3 + x^2 + \frac{1}{\lambda} x \right) \quad (0 < x \leq l_0) \quad (9)$$

Setting $D$ as the external diameter of the pipeline, the bending strain of the pipeline could be obtained according to the curvature of each point as:

$$\varepsilon(x) = \frac{D y''(x)}{2} = DM_B \left( \frac{3x^2}{2l} + \frac{1}{l^2} + \frac{1}{2\lambda l^3} \right) + \lambda x + 1 \right) \quad (0 < x \leq l_0) \quad (10)$$

### 3.2 Limiting condition of the pipe-soil coordinated deformation

The limiting condition of the pipe-soil coordinated deformation refers to the impending separation between a buried pipeline and the soil below the middle position of the pipeline in a subsidence zone. The condition is the transition state when the pipe-soil coordinated deformation stage begins to enter the partial hanging stage.

#### 3.2.1 Limiting criterion of the pipe-soil coordinated deformation

When the pipe-soil interaction satisfies the coordinated deformation condition, the pipeline fully bears the surface subsidence and transverse displacement of the surrounding soil, and the bending deformation of the buried pipeline matches the surface movement completely [Peng and Luo (1988)]. Assuming that $q$ represents the overlying load above the pipeline, and $r$ represents the radius of influence, then

$$y_{\text{max}} = qr^4/40\pi EI \quad (11)$$

#### 3.2.2 Pipeline failure criterion

The maximum strain criterion is introduced to analyze the failure of a buried pipeline. The failure of the buried pipeline occurs when the maximum strain $\varepsilon_{\text{max}}(x)$ is greater than the yield strain $\varepsilon_s$, namely, $\varepsilon_{\text{max}}(x) > \varepsilon_s$.

When the pipe-soil interaction is under the limiting condition of coordinated deformation and the pipeline failure criterion is tenable, the pipeline will fail in the coordinated deformation stage. Otherwise, the failure will occur in the partial hanging stage.

### 4 Mechanical analysis of a buried pipeline in the partially hanging stage

#### 4.1 Mechanical model

##### 4.1.1 Mechanical analysis of a buried pipeline in the coordinated deformation state

The partial hanging stage is shown in Figure 3, and the coordinate point in the critical hanging stage of the buried pipeline is $C (x_{co}, y_{\text{max}})$. In a subsidence zone,
the buried pipeline in the coordinated deformation state could be seen as the mechanical model of an elastic beam under uniform load. As the lateral load is the fourth-order derivative of the deflection, the beam deflection equation can be expressed as a fourth order polynomial:

$$y = b_4x^4 + b_3x^3 + b_2x^2 + b_1x + b_0 \quad (0 < x \leq x_{co})$$  \hspace{1cm} (12)

The deflection, slope, bending moment and shear force of the pipeline at point $B$ ($x = 0$) are continuous (figure 3), as:

$$y_{x=0} = 0, y'_{x=0} = \frac{M_B}{2EI\lambda}, y''_{x=0} = \frac{M_B}{EI}, y'''_{x=0} = \frac{\lambda M_B}{EI}$$  \hspace{1cm} (13)

The deflection of the pipeline at point $C$ ($x = x_{co}$) could be regarded as the maximum surface subsidence $y_{max}$. The angle $\theta$ ($C$) is same as the surface inclination $i^0$ ($x = x_{co}$) in the coordinated deformation state of the buried pipeline and the surrounding soil. Therefore, the expressions $M_B, b_0, b_1, b_2, b_3$ and $b_4$ are obtained as follows:

$$M_B = \frac{2EI(4y_{max}/x_{co}^4 - \lambda^2/4)}{2/(x_{co}^3 - 3x_{co}^2) + (1/2 - 2x_{co}) + (1/2 - 1)}$$

$$b_0 = 0, \quad b_1 = -\frac{M_B}{2EI\lambda}, \quad b_2 = -\frac{M_B}{2EI}, \quad b_3 = -\frac{\lambda M_B}{6EI}$$

$$b_4 = \frac{y_{max}}{x_{co}^4} + \frac{M_B}{2EI} \left( \frac{\lambda}{3x_{co}} + \frac{1}{x_{co}^2} + \frac{1}{\lambda x_{co}} \right)$$  \hspace{1cm} (14)

The expression of deflection $y$ is as follows:

$$y = \left( \frac{y_{max}}{x_{co}^4} + \frac{M_B}{2EI} \left( \frac{\lambda}{3x_{co}} + \frac{1}{x_{co}^2} + \frac{1}{\lambda x_{co}} \right) \right)x^4 - \frac{\lambda M_B}{6EI} x^3 + \frac{M_B}{2EI} x^2 + \frac{M_B}{2EI\lambda} x \quad (0 < x \leq x_{co})$$  \hspace{1cm} (15)

The bending strain of the pipeline could be obtained from the curvature of each point, as:

$$\varepsilon(x) = \frac{Dy''(x)}{2} = D \left( 6 \left( \frac{y_{max}}{x_{co}^4} + \frac{M_B}{2EI} \left( \frac{\lambda}{3x_{co}} + \frac{1}{x_{co}^2} + \frac{1}{\lambda x_{co}} \right) \right)x^2 - \frac{\lambda M_B}{2EI} x - \frac{M_B}{2EI} \right)$$

$$\quad (0 < x \leq x_{co})$$  \hspace{1cm} (16)

### 4.1.2 Mechanical analysis of a buried pipeline in the partial hanging stage

Assuming the length of the partial hanging pipeline is $l_1$, then $l_1 = 2(l_0 - x_{co})$. In the partial hanging stage, the mechanical analysis of a buried pipeline is shown in Figure 3. Note that the global coordinate system has been translated to the right,
that is, a local coordinate system $D - x_1 - y_1$ has been established with point $D$ $(x_{co},0)$ as the new origin (figure 3). The vertical and horizontal bending beam model is introduced in this study to perform the mechanical analysis of the partial hanging pipeline.

The bending differential equation of a pipeline in the partial hanging state is given as:

$$EI \frac{d^2 y_1}{dx_1^2} = N_{co}(y_1 - y_{co}) + \frac{1}{2} q x_1^2 - M_{co} - \frac{1}{2} q l_1 x_1$$

The solution is shown as follows:

$$y_1 = \frac{1}{N_{co}} \left( \frac{q EI}{N_{co}} - M_{co} \right) (\cosh(\sqrt{\frac{N_{co}}{EI}} x_1) - \tanh(\frac{l_1}{2} \sqrt{\frac{N_{co}}{EI}}) \sinh(\sqrt{\frac{N_{co}}{EI}} x_1) - 1) - \frac{q x_1}{2N_{co}} (x_1 - l_1) + y_{co}$$

(18)

When equation (18) is combined with the appropriate boundary conditions, deflection $y_1$ is obtained. Therefore, transforming the local coordinate to global coordinate,

$$y = \frac{1}{N_{co}} \left( \frac{q EI}{N_{co}} - M_{co} \right) (\cosh(\sqrt{\frac{N_{co}}{EI}} (x + x_{co})) - \tanh(\frac{l_1}{2} \sqrt{\frac{N_{co}}{EI}}) \sinh(\sqrt{\frac{N_{co}}{EI}} (x + x_{co})) - 1) - \frac{q(x + x_{co})}{2N_{co}} ((x + x_{co}) - l_1) + y_{co} \quad (x_{co} < x \leq l_0)$$

(19)
The corresponding bending strain of pipeline is shown as:
\[
\varepsilon(x) = \frac{Dy''(x)}{2} = \frac{D}{2} \left( \frac{M_{co} - q}{N_{co}} \right) \left[ \tanh \left( \frac{l_1}{2} \sqrt{\frac{N_{co}}{EI}} \right) \sinh \left( \sqrt{\frac{N_{co}}{EI}} (x + x_{co}) \right) 
- \cosh \left( \sqrt{\frac{N_{co}}{EI}} (x + x_{co}) \right) \right] \right)
- \frac{q(x + x_{co})}{4N_{co}} \left( q^2 (x + x_{co} - l_1) + 2 \right)
\]
\[(x_{co} < x \leq l_0) \tag{20}\]

### 4.2 Limiting condition of the pipe-soil in the partially hanging stage

#### 4.2.1 Limiting criterion of the pipeline in the partially hanging stage

In the pipe-soil partial hanging stage, the maximum surface subsidence \(W_0\) is given as:
\[
W_0(l_0) = W(l_0 + r) - W(l_0 + r - l) \tag{21}\]

The maximum subsidence \(f\) of buried pipeline is given as:
\[
f = \frac{ql_1^2}{8N_{co}} - \frac{1}{N_{co}} \left( \frac{qEI}{N_{co}} - M_{co} \right) \left( (\cosh \left( \frac{l_1}{2} \sqrt{\frac{N_{co}}{EI}} \right) - 1) / \cosh \left( \frac{l_1}{2} \sqrt{\frac{N_{co}}{EI}} \right) \right) + y_{co} \tag{22}\]

In the limiting condition of pipe-soil partial hanging stage, the maximum subsidence of buried pipeline is equal to the maximum surface subsidence. Thus, from equations (21) and (22),
\[
W(l_0 + r) - W(l_0 + r - l) = \frac{ql_1^2}{8N_{co}} - \frac{1}{N_{co}} \left( \frac{qEI}{N_{co}} - M_{co} \right) \left( (\cosh \left( \frac{l_1}{2} \sqrt{\frac{N_{co}}{EI}} \right) - 1) / \cosh \left( \frac{l_1}{2} \sqrt{\frac{N_{co}}{EI}} \right) \right) + y_{co} \tag{23}\]

Hence, the advancement length \(l\) of the working face in the pipe-soil partial hanging limit criterion could be obtained as shown in equation (23).

#### 4.2.2 Failure criterion for the pipeline

When the maximal strain value \(\varepsilon_{\text{max}}(x)\) is greater than the yield strain \(\varepsilon_s\), namely \(\varepsilon_{\text{max}}(x) > \varepsilon_s\), the failure of the buried pipeline occurs. If the failure criterion for the pipeline is tenable when the pipe-soil interaction is in partial hanging limit condition, the pipeline fails in the pipe-soil partial hanging stage.

### 5 A case study in the field

#### 5.1 Project background

A gas pipeline route is considered to demonstrate the practical application of the analysis method proposed in this study. The pipeline route passes through the third
mining district of a coal mine in Shanxi Province, China and is parallel to work face strike #14301 of that mining district, as shown in Figure 5.

![Figure 5: Distribution of a shallow-buried pipeline in a mine.](image)

The mining parameters of work face #14301 in the third mining district are shown in Table 1. The material of buried pipeline is API 5L X60, and the corresponding parameters are shown in Table 2.

<table>
<thead>
<tr>
<th>buried depth of coal seam (m)</th>
<th>thickness of coal seam (m)</th>
<th>dip angle of coal seam (°)</th>
<th>strike length of working face (m)</th>
<th>inclined length of working face (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>580/ (m)</td>
<td>7/ (m)</td>
<td>1~3/ (°)</td>
<td>2350/ (m)</td>
<td>215/ (m)</td>
</tr>
</tbody>
</table>

5.2 The analysis of the influence of coal mining on the buried pipeline

The analysis of the influence of coal mining on the buried pipeline needs to analyze the mechanical response of the buried pipeline, first in the pipe-soil coordinated deformation stage and, then, in the partial hanging stage. The mechanical analysis of the buried pipeline in the pipe-soil coordinated deformation stage is shown in
Table 2: Parameters of buried pipeline.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>buried depth (m)</td>
<td>1.4</td>
</tr>
<tr>
<td>outer radius (mm)</td>
<td>660</td>
</tr>
<tr>
<td>nominal thickness (mm)</td>
<td>7.1</td>
</tr>
<tr>
<td>density of pipeline (kg/m³)</td>
<td>7.8 × 10³</td>
</tr>
<tr>
<td>elasticity modulus (GPa)</td>
<td>205</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>yield strain</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Figure 6; a similar flow chart is applicable to the buried pipeline in the pipe-soil partial hanging stage as well.

Figure 6: Flow chart of coordinated deformation stage.

(1) Pipe-soil coordinated deformation stage

After putting the parameters of the pipeline and the surrounding soil into pipe-soil coordinated deformation limit criterion, namely equation (11), the maximum surface subsidence in the limiting condition of the pipe-soil coordinated deformation was obtained as 2.6 m. Then, the probability integral method was introduced to
calculate the mining length of the working face, which was obtained as 435 m. Putting the related calculated parameters into equation (10), the maximum strain of the buried pipeline was calculated as 0.034, which is less than yield strain of the pipeline (0.04). Therefore, it can be concluded that the buried pipeline is safe. With the advancement of the working face, the pipe-soil partial hanging stage will be realized.

(2) Pipe-soil partial hanging stage
After putting the parameters of the pipeline and the surrounding soil into the pipe-soil partial hanging limit criterion, namely equation (23), the mining length of working face is obtained as 682 m in the limiting condition of the partial hanging pipeline, and the maximum surface subsidence as 5.5 m. Furthermore, putting the related calculated parameters into equation (20), the maximum strain of the buried pipeline is calculated as 0.055, which is larger than yield strain of pipeline (0.04). Therefore, it can be concluded that failure of buried pipeline occurs in the partial hanging stage.

In summary, the interaction between a buried pipeline and its surrounding soil should be controlled within the coordinated deformation stage, in which the safe maximum length of the working face should be limited to 435 m.

6 Conclusions

(1) The interaction between a buried pipeline and its surrounding soil in mining is classified into two stages, namely, coordinated deformation stage and partial hanging stage. a) the pipeline in non-mining subsidence zone, b) the pipeline at the coordinated deformation stage, and c) the pipeline at the hanging stage are mechanically analyzed with the models of a) a beam on an elastic foundation, b) an elastic beam under uniform load, and c) a vertical and horizontal bending beam, respectively. The mechanical model of segmented elastic beam in this study simulated the pipe-soil interaction during mining underneath the pipe.

(2) By analyzing the mechanical response of a buried pipeline in the limiting condition of pipe-soil interaction in each stage, the limiting criterion and the failure criterion of a buried pipeline were established for each stage. If the buried pipeline is in the limiting state but the failure criterion for the pipeline has not been satisfied, the failure may occur only in the next stage.

(3) The mining practice under a gas pipeline in Shanxi province, China was analyzed, resulting to the conclusion that the mode of failure for the buried pipeline will likely be in the partial hanging stage. The pipe-soil interaction should, therefore, be controlled within the coordinated deformation stage during coal mining. The safe maximum length of the working face of the coal mining was determined
as 435 m by the method proposed in this study.

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**References**


