Hybrid Adaptive Particle Swarm Optimized Particle Filter for Integrated Navigation System

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Abstract: Particle swarm optimization algorithm based particle filter is trapping in local optimum easily, it is not able to satisfy the requirement of modern integrated navigation system. In order to solve the problem, A novel particle filter algorithm based on hybrid adaptive particle swarm optimization (HPSO-PF) is presented in this paper. This improved particle filter will conduct to finding the ideal solution domain by making use of the global convergence of artificial fish swarm and enhancement of fusion precision by guiding particles to move toward the high likelihood area through particle swarm optimization. Finally different models are used for simulation and the experiment results show that this new particle filter improves the precision of integrated navigation system.

Keywords: dynamic, particle filter, integrated navigation, hybrid adaptive.

1 Introduction

The measurement error of global positioning system is not accumulated over the time, but the system is prone to disturbance despite the high measurement accuracy [e.g. Bhatt, Aggarwal and Devabhaktuni (2012)]. By comparison, however, integrated navigation system is free from external disturbance due to the use of independent navigation mode, but the measurement error is accumulated over time [e.g. Soon, Scheding and Lee (2008)]. GPS/INS integrated navigation system has upgraded the overall performance significantly [e.g. Hu, Gao and Zhong (2015)] Provided global positioning system receiver is capable of receiving the information sent by at least 4 satellites, GPS can offer solution position information, and inertial navigation system will measure the position and attitude information of the aircraft by means of angular rate sensor and linear acceleration.

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In the indirect estimation on the integrated navigation system, navigation parameter error equation constitutes the main part of integrated navigation system’s state equation. However, considering that there is a small error, the rules of integrated navigation parameter error will be described by Kalman filter and first-order approximation equation in case of the requirement for low accuracy and the model error is not high.

However, we have made an increasingly higher demand [e.g. Cho (2014)] on the accuracy of integrated navigation in recent years, because of which the model error resulting from the use of low-order approximation cannot be ignored. Additionally, measurement noise and system noise may be the non-Gaussian noises Particle filter (PF) [e.g. Zhou, Yang, and Mi (2012)] is a statistic filtering based on Monte Carlo Method it is widely applied to positioning and navigation of non-linear system and non-Gaussian noise field [e.g. Wang Haynes Huang Dong and Atluri (2015)] as its state function and observation function has no non-linear and non-Gaussian hypothesis. Nevertheless, particle filter may confront with the problem of weight degradation which if solved by resampling method may result unavoidable particle impoverishment.

Particle filter based on intelligent optimization conduces to significant improvement of particle degradation in Particle filter and great enhancement of precision. Particle filter based on particle swarm optimization (PSO-PF) is a typical representative of intelligent optimized particle filter which introduces particle swarm optimization into particle filter. Through introduction of the latest observation value to the sample distribution, along with the utilization of particle filter for sampling process optimization and constantly update of particle speed, the sample distribution is inclined to move to the true area with higher posterior probability. Particle filter based on particle swarm optimization improves the particle degradation of article filter. However, PSO-PF is a process of iterative optimization which will prolong the calculation time because of the high iterative frequency, and thus is hard to meet the needs of real-time target tracking by radar in actual practices. Moreover, particle filter based on particle swarm optimization will be easily trapped into local optimization [e.g. Chen, Bo Wu and Zhou (2013)].

This paper presents a new hybrid adaptive optimization algorithm based on artificial fish swarm algorithm (AFSA) and particle swarm optimization algorithm. This new algorithm finds satisfied particle range by artificial fish swarm, and then performs quick local searching by PSO, accordingly updating the information of corrected particles and enhanced the local searching speed of particles and showing global convergence property In this way, the locating precision of integrated navigation system was improved.
2 Particle filter

Particle filter (PF) is an approximate calculation of Bayes estimation based on sampling theory. Combining Monte Carlo Method and Bayesian Theory, PF follows the thought that to gain a group of sample for approximation of posterior probability density, replace the infinitesimal calculus in light of posterior probability density function by sample mean value, and thus acquire the minimal estimate of variance [e.g. Jian Xu and Yin (2010)].

3 AFSA algorithm

Artificial fish swarm algorithm is an optimization algorithm by simulation of behaviors of shoals of fish swarm, which starts from behaviors of the animals in simple structures and ends up with highlighting global optimized value in the group by means of optimization for behaviors of individual artificial fish. The algorithm is described as follows: Assuming the state of individual artificial fish be expressed as vector \( X = (x_1, x_2, \cdots, x_n) \), wherein \( x_i (i = 1, \cdots, n) \) signifies variables to be optimized; the density of artificial fish is localized is \( Y = f(X) \), wherein, \( Y \) indicates the objective function value; the distance between individual artificial fish is expressed as \( d_{i,j} = \|X_i - X_j\| \), and \textit{visual} represents the conception range for artificial fish; \( \delta \) is congestion factor; and step is expressed as the length of moving step. The behavior of artificial fish is described as follows:

(a) Foraging behavior. Upon detection of food, artificial fish would swim towards the gradually-increasing food areas. Set the current state of fish is \( X_i \), randomly select a state \( X_j \) within the conception range \( (d_{i,j} < \text{\textit{visual}}) \), and calculate the fitness function. When \( Y_i < Y_j \), \( X_i \) steps towards \( X_j \); otherwise with random step-forward, a new state will be entered into, i.e.:

\[
\text{If } Y_i < Y_j, X_{\text{inext}} = X_i + r \cdot s \cdot \frac{X_j - X_i}{\|X_i - X_j\|}; \text{ Else } X_{\text{inext}} = X_i + r \cdot s
\]

Wherein, \( s \) is the maximum step length; \( r \) indicates a random number between 0 and 1.

(b) Swarm behavior. To ensure their own survivals free from harmfulness, artificial fish naturally swarms together. Fish \( X_i \) searches for the numbers of partners \( n_f \) and central position \( X_c \) within the field of view. When \( Y_c/n_f > \delta \cdot Y_i \), there is adequate food among the artificial fish and it would not be so packed, thus \( X_i \) steps forward; else, perform foraging behavior.

\[
\text{If } Y_c/n_f > \delta \cdot Y_i, \quad X_{\text{inext}} = X_i + r \leq s \cdot \frac{X_c - X_i}{\|X_c - X_j\|}; \quad (2)
\]
4 PSO-PF algorithm

The sampling process of particle filter is suboptimal, whereas the incorporation of particle swarm optimization can optimize the sampling process of particle filter, allow the weight sets are more inclined to high likelihood area [e.g. Hwang Sung (2013)], accordingly solving the problem of impoverishment, and conducing to reduction of particle numbers required by particle filter. particle swarm optimization thought is fused with particle and the key lies in utilizing the optimized state value $P_{pbest}$ experienced by the particles and the state value $P_{gbest}$ of the maximum particle with the greatest fitness function value, and updating the particles’ speed and position through equation (3) and (4), accordingly forcing the particles to be closer to the real state.

$$V^i_k = |Randn| \times (P_{pbest}^i - X^i_{k-1}) + |randn| \times (P_{gbest}^i - X^i_{k-1})$$

(3)

$$X^i_k = X^i_{k-1} + V^i_{k-1}$$

(4)

Where $|Randn|$ and $|randn|$ are positive Gaussian distribution random numbers.

5 Building of GPS/INS Integrated Navigation Model

5.1 State and Measurement Equations

The application of particle filter to integrated navigation system is ultimately intended for a more accurate parameter [e.g. Chen Z.M, Bo Wu and Yu (2012)], and the selection of filter state normally resorts to indirect process, i.e., the error $\Delta X$ of navigation parameter outputted by a certain system is taken as particle filter’s estimated value. While indirect process is used for estimation, the estimated state of PF will be the combination of various errors in integrated navigation system. Therefore, the estimation process of PF should be independent of the computation of the integrated navigation parameters, and the INS will have the strength of high update frequency fully revealed [e.g. Hide Moore and Smith (2003)].

Supposing the combination mode of integrated navigation system relies on the combination between attitude and velocity, integrated navigation system’s measurement can be divided into two values, namely, difference value of position and that of velocity [e.g. Ding, Wang and Rizos (2007)]. Difference value of position means that the difference between the position information rendered by inertial navigation system and the information of relevant position calculated by global positioning system receiver is figured out as measurement information. Difference value of velocity measurement means that the difference between the information rendered by inertial navigation system and the information of relevant velocity offered by
global positioning system receiver is worked out as another type of measurement information.

The error state equation of integrated navigation system is shown as follows:

$$
X(t) = F(t)X(t) + G(t)W(t)
$$

(5)

Where, \(X(t) = [\phi_E \phi_N \phi_U \delta v_E \delta v_N \delta v_U \delta \lambda \delta h \varepsilon_{bE} \varepsilon_{bN} \varepsilon_{bU} \varepsilon_{rx} \varepsilon_{ry} \varepsilon_{rz} \nabla x \nabla y \nabla z]^T_{18 \times 1}\)

The position measurement of inertial navigation system will be expressed as the sum of true value and corresponding error under the geographic coordinate system.

$$
\begin{bmatrix}
L_l \\
\lambda_l \\
h_l
\end{bmatrix} = 
\begin{bmatrix}
L_t + \delta L \\
\lambda_t + \delta \lambda \\
h_t + \delta h
\end{bmatrix}
$$

(6)

The position measurement offered by global positioning system receiver can be expressed as the difference between true value and corresponding error under the geographic coordinate system.

$$
\begin{bmatrix}
L_G \\
\lambda_G \\
h_G
\end{bmatrix} = 
\begin{bmatrix}
L_t - \frac{N_e}{R_N} \\
\lambda_t - \frac{N_N}{R_N \cos L} \\
h_t - N_h
\end{bmatrix}
$$

(7)

Where \(\lambda_t, L_t,\) and \(h_t\) are actual location, and \(N_E, N_N,\) and \(N_U\) for the errors of global positioning system receiver in the eastward, northward and skyward directions.

The position measurement vector is defined as follows:

$$
Z_p(t) = 
\begin{bmatrix}
(L_l - L_G)R_M \\
(\lambda_l - \lambda_G)R_N \cos L \\
h_l - h_g
\end{bmatrix} = 
\begin{bmatrix}
R_M \delta L + N_N \\
R_N \delta \lambda \cos L + N_E \\
\delta h + N_U
\end{bmatrix} \equiv H_p(t)X(t) + V_p(t)
$$

(8)

Where, \(H_p = \begin{bmatrix} 0_{3 \times 6} & \text{diag}[R_M R_N \cos L 1] & 0_{3 \times 9} \end{bmatrix}_{3 \times 18}\)

\(V_p = [N_N N_E N_U]^T\)

The variances of measurement noise are \(\sigma_{pN}^2, \sigma_{pE}^2,\) and \(\sigma_{pU}^2.\)

$$
\begin{align*}
\sigma_{pN} &= \sigma_p \cdot HDOP_N \\
\sigma_{pE} &= \sigma_p \cdot HDOP_E \\
\sigma_{pU} &= \sigma_p \cdot HDOP_U
\end{align*}
$$

(9)

Where, \(\sigma_p\) stands the pseudo-range measurement error.
The velocity measurement information of inertial navigation system can be expressed as the sum of true value and corresponding velocity error under the geographic coordinate system.

\[
\begin{bmatrix}
  v_{IN} \\
  v_{IE} \\
  v_{IU}
\end{bmatrix} =
\begin{bmatrix}
  v_N + \delta v_N \\
  v_E + \delta v_E \\
  v_U + \delta v_U
\end{bmatrix}
\] (10)

Where \( v_E, v_N, \) and \( v_U \) represent the true velocities along eastward, northward and skyward axes under geographic coordinate system.

The velocity measurement of global positioning system can be also expressed as the difference between true value and corresponding velocity measurement error under the geographic coordinate system

\[
\begin{bmatrix}
  v_{GN} \\
  v_{GE} \\
  v_{GU}
\end{bmatrix} =
\begin{bmatrix}
  v_N - M_N \\
  v_E - M_E \\
  v_U - M_U
\end{bmatrix}
\] (11)

Where \( M_N, M_E, M_U \) constitute the components of velocity measurement errors of GPS receiver along three axes, namely, northward, eastward and skyward axes.

Below is the definition of velocity measurement vector:

\[
Z_p(t) =
\begin{bmatrix}
  v_{IN} - v_{GN} \\
  v_{IE} - v_{GE} \\
  v_{IU} - v_{GU}
\end{bmatrix} =
\begin{bmatrix}
  \delta v_N + M_N \\
  \delta v_E + M_E \\
  \delta v_U + M_U
\end{bmatrix} \equiv H_v(t)X(t) + V_v(t) \quad (12)
\]

Where, \( H_v = \begin{bmatrix} 0_{3 \times 3} & \text{diag}[1 \ 1 \ 1] & 0_{3 \times 12} \end{bmatrix} \), \( V_v = [M_N \ M_E \ M_U]^T \)

Assuming the measurement velocity of pseudo-range rate \( \dot{\rho} \) of global positioning system receiver is \( \sigma_p^2 \), the standard deviations of the eastward, northward and skyward velocity errors resulting from pseudo-range rate is:

\[
\begin{align*}
\sigma_{vE} &= H\text{DOP}_E \cdot \sigma_p \\
\sigma_{vN} &= H\text{DOP}_N \cdot \sigma_p \\
\sigma_{vU} &= V\text{DOP} \sigma_p
\end{align*}
\] (13)

The combination of position vector with velocity vector can obtain the measurement equation of position and speed integration system:

\[
Z(t) =
\begin{bmatrix}
  Z_p(t) \\
  Z_v(t)
\end{bmatrix} =
\begin{bmatrix}
  H_p \\
  H_v
\end{bmatrix} X(t) +
\begin{bmatrix}
  V_p(t) \\
  V_v(t)
\end{bmatrix} = H(t)X(t) + V(t) \quad (14)
\]
5.2 Discretization of State and Measurement Equations

Following result can be obtained through the state equation (3) and measurement equation (14):

\[ X_k = \Phi_{k,k-1}X_{k-1} + \Gamma_{k-1}W_{k-1} \]  

\[ Z_k = H_kX_k + V_k \]

Where, \( \Phi_{k,k-1} = \sum_{n=0}^{\infty} [F(t_n)T]^n/n! \), \( \Gamma_{k-1} = \left\{ \sum_{n=1}^{\infty} \frac{1}{n!}[F(t_k)T]^{n-1} \right\} G(t_k)T \)

As required by filter, the system and measurement noises of state and measurement equations should be equipped with following characteristics:

\[ E\{W(t)W^T(\tau)\} = Q(t)\delta(t-\tau) \]  

\[ E\{V(t)V^T(\tau)\} = R(t)\delta(t-\tau) \]  

\[ E\{W_kW_j^T\} = Q_k\delta_{kj} \]  

\[ E\{V_kV_j^T\} = R_k\delta_{kj} \]

Where, \( \delta_{kj} = \begin{cases} 1 & k = j \\ 0 & k \neq j \end{cases} \)  

\( Q_k = Q(t)/T \)  

\( R_k = R(t)/T \).

6 HPSO-PF Algorithm

HPSO-PF searches for optimization by use of hybrid intelligent optimization algorithm to obtain the high likelihood value.

The algorithm steps are listed as follows:

(1): When \( k=0 \), take \( N \) particles \( \{x_{0,k}^i, i = 1,...,N\} \) as samples from importance function at the initial time. The importance density function is expressed in equation (21):

\[ x_k^i \sim q(x_k^i|x_{k-1}^i, z_k) = p(x_k^i|x_{k-1}^i) \]  

Where \( p(\cdot) \) is probability density function, \( q(\cdot) \) is importance density function [e.g. Ding, Wang and Rizos (2007); Wang and Qian (2012); Zhang Xin and Yang (2013); Yang and Sun (2013)].

Fitness function:

\[ Y = \exp[-\frac{1}{2R_k}(z_{New} - z_{Pred})] \]
Where $R_k$ represents observation noise, $z_{New}$ represents the latest observation value, $z_{Pred}$ represents the estimated observation value.

(2): Calculate the weight:

$$w_k = w_{k-1}^j p(z_k|x_{k-1}^j) = w_{k-1}^j \frac{p(z_k|x_{k-1}^j)p(x_{k}^i|x_{k-1}^j)}{q(x_k^i|x_{k-1}^j,z_k)} = w_{k-1}^j p(z_k|x_{k-1}^i)$$

\[ \tag{23} \]

(3) Make random initialization within feasible area for the size of fish swarm $N$, initial position of each fish, vision is visual, crowded degree factor is $\delta$, maximum repeated attempts is trynummber, acceleration factors is $c_1$ and $c_2$.

(4) Calculate the fitness of each fish, and make comparisons with the state on bulletin board. If the former state is better, then assign it onto the bulletin board.

(5) Each fish updates its own position through foraging behavior, clustering behavior, piling up behavior, and random behavior.

(6) Artificial fish swarm termination conditions. In case of attainment of preset evolution algebra, then update the optimized value. Turn to step (7), or else step (4).

(7) Re-initialize the position and velocity of each artificial fish, or assign particle information upon maximum evolution algebra of artificial fish to swarm particle information.

(8) Assign the optimized position and value information on the bulletin board to $p_{best}$ and $g_{best}$.

(9) Evaluate the fitness of each artificial fish.

(10) Make comparisons of the fitness function value with that in the optimized position $p_{best}$ of each particle [e.g. Zhang Xin and Yang (2012); Xian Long and Li (2014); Li Bai and Zhang (2010); MirHassani and Abolghasemi (2011)]. If the former is better, take it as the currently optimized position $p_{best}$.

(11) Make comparisons of the fitness value with that in the globally optimized position $p_{best}$ of each particle. If the former is better, update the globally optimized position $g_{best}$.

(12) Update the speed and position of artificial fish according to formula (16) and (17).

(13): When the optimized value of particle complies with the initially-set threshold value $\varepsilon$, it is indicated that the particles have been already distributed around the true values, the initially-set threshold value is set as 0.15. Additionally, when the improved algorithm reaches the maximum iteration number, the iteration will be terminated. In the simulation, the maximum number of iterations is set as 25. By now particle optimization should be stopped. Or $E_t = E_{t-1}, G_t = G_{t-1}$, and execute
step (4).

(14): Calculate the importance weight of the particles after optimization and perform normalization.

\[ w_k^i = w_k^i / \sum_{i=1}^{N} w_k^i \]  \hspace{1cm} (24)

(15): State output:

\[ \tilde{x} = \sum_{i=1}^{N} w_k^i x_k^i \]  \hspace{1cm} (25)

7 Experimental simulation

7.1 Simulation test of basic algorithm performance

Choosing a univariate nonstationary growth model, and the process model and measurement model are given as follows:

\[ x(t) = 0.5x(t-1) + \frac{25x(t-1)}{1 + [x(t-1)]^2} + 8\cos[1.2(t-1)] + w(t) \]  \hspace{1cm} (26)

\[ z(t) = \frac{x(t)^2}{20} + v(t) \]  \hspace{1cm} (27)

Where, \( w(t) \) and \( v(t) \) are zero-mean Gaussian noise. This system is highly nonlinear and the likelihood function presents bimodal [e.g. Li and Wang (2012)].

\[ \eta = \frac{rSTEP - temp}{rSTEP} \]  \hspace{1cm} (28)

Where, \( \eta \) is the optimization success rate, calculation marking variable is \( temp \), sampling time is \( rSTEP \).

By using PF, PSO-PF, HPSO-PF, state estimation and tracking of this non-linear system are performed, and the formula of root-mean-square error is

\[ RMSE = [\frac{1}{T} \sum_{t=1}^{T} (x_t - \hat{x}_t)^2]^{1/2} \]  \hspace{1cm} (29)

(1) Giving the number of particles \( N = 100 \), and process noise variance \( Q = 10 \), measurement noise variance \( R = 1 \), the simulation result is presented in figure 1 and figure 2. After 200 times of Monte-Carlo simulation, the result is given in Table 1.

(2) Giving the number of particles \( N = 100 \), and process noise variance \( Q = 20 \), measurement noise variance \( R = 1 \), the simulation result is presented in figure 3 and figure 4. After 500 times of Monte-Carlo simulation, the result is given in Table 1.
Figure 1: State estimation of different algorithm ($Q = 10$).

Figure 2: RMSE of different algorithm ($Q = 10$).
Figure 3: State estimation of different algorithm ($Q = 20$).

Figure 4: RMSE of different algorithm ($Q = 20$).
Table 1: Comparison of simulation parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Algorithms</th>
<th>$\eta$ /%</th>
<th>RMSE</th>
<th>Operation time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q = 10, N = 200$</td>
<td>PF</td>
<td>/</td>
<td>3.5892</td>
<td>0.6684</td>
</tr>
<tr>
<td>$Q = 10, N = 50$</td>
<td>PSO-PF</td>
<td>97.83</td>
<td>2.4563</td>
<td>0.5821</td>
</tr>
<tr>
<td>$Q = 10, N = 50$</td>
<td>HPSO-PF</td>
<td>99.34</td>
<td>1.4663</td>
<td>0.5033</td>
</tr>
<tr>
<td>$Q = 20, N = 200$</td>
<td>PF</td>
<td>/</td>
<td>5.9635</td>
<td>0.6854</td>
</tr>
<tr>
<td>$Q = 20, N = 50$</td>
<td>PSO-PF</td>
<td>97.72</td>
<td>4.2155</td>
<td>0.5915</td>
</tr>
<tr>
<td>$Q = 20, N = 50$</td>
<td>HPSO-PF</td>
<td>98.79</td>
<td>2.5182</td>
<td>0.5367</td>
</tr>
</tbody>
</table>

As shown by the experimental result, the error of the integration with particle swarm optimized particle filter is significantly lower than that of PF, and the integration is in fact the particle optimization process of particle swarm optimization that can improve particle quality. The algorithm use the global convergence of artificial fish swarm algorithm and the local convergence of particle swarm optimization the quality of particle swarm was heightened.

7.2 Simulation test of performance in integrated navigation system

Let the latitude and longitude of the initial position of state vector be 32° and 118°, respectively; the random and constant drift errors of the gyroscope be 0.05°/h respectively; the random and constant bias errors of the gyroscope be 50 µg and 100 µg, respectively; the update cycle of inertial navigation be 0.01 s; the cycle of Kalman filtering be 1 s; and the simulation time be 500 s. In this paper, an analysis is implemented on the position and velocity error curves along northward, eastward and skyward directions before and after the integrated filter correction, 500 times simulations have been made to obtain the average value.

As illustrated by the figure, the system faces rapid divergence prior to the application of integrated filter, but the parameter errors of the system are correctly effectively upon the use of improved particle filter algorithm. The reason thereof is that although the current algorithm may lead to more significant errors when locating for satisfied value range due to interference from environmental factors during the process of global optimization in artificial fish swarm algorithm stage, in the afterward particle swarm optimization stage, particles can constantly update their values through individual information and swarm information, thus improving the accuracy of system.
Figure 5: Position error in different directions (northward, eastward, skyward).

Figure 6: Velocity error in different directions (northward, eastward, skyward).
8 Conclusion

This paper, by integrating artificial fish swarm algorithm together with particle swarm optimization for application to integrated navigation model, and by use of desirable global convergence of AFSA in addition to the fastness of convergence with PSO, enjoys higher searching efficiency and convergence accuracy as compared to the previous methods The experimental results show that the improved algorithm in this paper improves the precision and thus of high applicable value in GPS/INS integrated navigation system.

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