MHD Natural Convection in a Nanofluid-filled Enclosure with Non-uniform Heating on Both Side Walls

Imen Mejri\textsuperscript{1,2}, Ahmed Mahmoudi\textsuperscript{1}, Mohamed Ammar Abbassi\textsuperscript{1} and Ahmed Omri\textsuperscript{1}

Abstract: This study examines natural convection in a square enclosure filled with a water-Al\textsubscript{2}O\textsubscript{3} nanofluid and subjected to a magnetic field. The side walls of the cavity have spatially varying sinusoidal temperature distributions. The horizontal walls are adiabatic. A Lattice Boltzmann method (LBM) is applied to solve the governing equations for fluid velocity and temperature. The following parameters and related ranges are considered: Rayleigh number of the base fluid, from Ra=10\textsuperscript{3} to 10\textsuperscript{6}, Hartmann number from Ha=0 to 90, phase deviation (γ=0, π/4, π/2, 3π/4 and π) and solid volume fraction of the nanoparticles between φ=0 and 6%. The results show that the heat transfer rate increases with an increase in the Rayleigh number but it decreases with an increase in the Hartmann number. For γ=π/2 and Ra=10\textsuperscript{5} the magnetic field strengthens the effect produced by the presence of nanoparticles. For Ha=0, the most evident influence of nanoparticles is achieved at γ=0 and π/4 for Ra=10\textsuperscript{4} and 10\textsuperscript{5} respectively.

Keywords: Lattice Boltzmann Method, Natural convection, nanofluid, magnetic field, Sinusoidal temperature distribution.

Nomenclature

\begin{tabular}{ll}
\textbf{Symbol} & \textbf{Definition} \\
\hline
B & Magnetic field (T) \\
c & Lattice speed (m/s) \\
c_s & Speed of sound (m/s) \\
c_i & Discrete particle speeds (m/s) \\
c_p & Specific heat at constant pressure (JK\textsuperscript{-1}K\textsuperscript{-1}) \\
F & External forces (N) \\
f & Density distribution functions (kg m\textsuperscript{-3}) \\
\end{tabular}

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<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$f^{eq}$</td>
<td>Equilibrium density distribution functions ($\text{kg m}^{-3}$)</td>
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<tr>
<td>$g$</td>
<td>Internal energy distribution functions ($\text{K}$)</td>
</tr>
<tr>
<td>$g^{eq}$</td>
<td>Equilibrium internal energy distribution functions ($\text{K}$)</td>
</tr>
<tr>
<td>$\vec{g}$</td>
<td>Gravity vector ($\text{m s}^{-2}$)</td>
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<tr>
<td>Ha</td>
<td>Hartmann number</td>
</tr>
<tr>
<td>$k$</td>
<td>Thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)</td>
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<tr>
<td>Ma</td>
<td>Mach number</td>
</tr>
<tr>
<td>n</td>
<td>Number of nodes</td>
</tr>
<tr>
<td>Nu</td>
<td>Local Nusselt number</td>
</tr>
<tr>
<td>P</td>
<td>Pressure ($\text{N m}^{-2}$)</td>
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<td>Pr</td>
<td>Prandtl number</td>
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<tr>
<td>Ra</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>T</td>
<td>Temperature ($\text{K}$)</td>
</tr>
<tr>
<td>$u(u,v)$</td>
<td>Velocities ($\text{m/s}$)</td>
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<tr>
<td>$x(x,y)$</td>
<td>Lattice coordinates ($\text{m/s}$)</td>
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**Greek symbols**

<table>
<thead>
<tr>
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<tr>
<td>$\Delta x$</td>
<td>Lattice spacing ($\text{m}$)</td>
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<tr>
<td>$\Delta t$</td>
<td>Time increment ($\text{s}$)</td>
</tr>
<tr>
<td>$\tau_{\alpha}$</td>
<td>Relaxation time for temperature ($\text{s}$)</td>
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<tr>
<td>$\tau_{v}$</td>
<td>Relaxation time for flow ($\text{s}$)</td>
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<tr>
<td>$\nu$</td>
<td>Kinematic viscosity ($\text{m}^2 \text{s}^{-1}$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity ($\text{m}^2 \text{s}^{-1}$)</td>
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<tr>
<td>$\rho$</td>
<td>Fluid density ($\text{kg m}^{-3}$)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Electrical conductivity ($\text{S/m}$)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Non-dimensional stream function</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Solid volume fraction</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity ($\text{N s/m}^2$)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Phase deviation</td>
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<tr>
<td>$\theta$</td>
<td>Non-dimensional temperature</td>
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**Subscript**

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<tr>
<td>b</td>
<td>Bottom</td>
</tr>
<tr>
<td>c</td>
<td>Cold</td>
</tr>
<tr>
<td>f</td>
<td>Fluid</td>
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<tr>
<td>h</td>
<td>Hot</td>
</tr>
<tr>
<td>l</td>
<td>Left</td>
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1 Introduction

The problem of natural convection in square enclosures has many engineering applications such as: cooling systems of electronic components, building and thermal insulation systems, built-in-storage solar collectors, nuclear reactor systems, food storage industry and geophysical fluid mechanics and many others [Ostrach (1988); Al-Ajmi and Mosaad, (2012); Hamimid, Guellal, Amroune and Zeraibi, (2012); Moufekkir, Moussaoui, Mezrhab, Naji and Bouzidi, (2012); Choukairy and Benmecer, (2012); Arid, Kousksou, Jegadheeswaran, Jamil, Zeraouli, (2012); Dihmani, Amraqui, Mezrhab and Laraqi, (2012); Shemirani and Saghiri, (2013); Maougal and Bessaïh, (2013); Kamath, Balaji and Venkateshan, (2013); Ribi, Hasnaoui and Amahmid, (2013); Mahrouche, Najam, El Alami, Faraji, (2013); Rana and Thakur, (2013); Haslavsky, Miroshnichenko, Kit, and Gelfgat, (2013)]. In some practical cases such as crystal growth, metal casting, fusion reactors and geothermal energy extractions, natural convection is under the influence of a magnetic field [Moreau (1990); Ozoe and Okada (1989); Garandet et al. (1992); Venkatachalappa and Subbaraya (1993); Alchaar et al. (1995); Rudraiah et al. (1995)]. Khanafer et al. (2003) numerically investigated natural convection heat transfer in a two-dimensional vertical enclosure utilizing nanofluids. It was revealed that the heat transfer rate increases with the increase of particle fraction at any given Grashof number. Kahveci (2010) numerically studied the heat transfer enhancement of water-based nanofluids in a differentially heated, tilted enclosure for a range of inclination angles, nanoparticle volume fractions, and Rayleigh numbers. It was concluded from the results that suspended nanoparticles substantially increase the heat transfer rate and the average Nusselt number is nearly linear with the increase of solid volume fraction. However, Putra et al. (2003) conducted experiments to investigate natural convective heat transfer of aqueous CuO and Al₂O₃ nanofluids inside a cylinder. They observed a systematic and significant deterioration in natural convective heat transfer at Rayleigh numbers from 10⁶ to 10⁹. The deterioration increased with particle concentration and was more pronounced for CuO nanofluids. Wen and Ding (2005) reported that for a Rayleigh number less than 10⁶, the natural convection heat transfer rate increasingly decreases with the increase of particle fraction, particularly at low Rayleigh numbers. Pirmohammadi and Ghassemi (2009) studied steady laminar natural-convection flow in the presence of a mag-
netic field in a tilted enclosure heated from below and cooled from top and filled with liquid gallium. They found that for a given inclination angle, as the value of Hartmann number increases, the convection heat transfer is reduced. Furthermore they obtained that at Ra=$10^4$, the value of Nusselt number depends strongly on the inclination angle for relatively small values of Hartmann number.

Ece and Buyuk (2006) examined the steady and laminar natural convection flow in the presence of a magnetic field in an inclined rectangular enclosure heated and cooled on its adjacent walls. They found that the magnetic field suppressed the convective flow and the heat transfer rate. They also showed that the orientation and the aspect ratio of the enclosure and the strength and direction of the magnetic field had significant effects on the flow and temperature fields. Sathiyamoorthy and Chamkha (2010) numerically studied natural convection flow of electrically conducting liquid gallium in a square cavity whereas the bottom wall is uniformly heated and the left and right vertical walls are linearly heated while the top wall is kept thermally insulated. They exhibited that the magnetic field with inclined angle has effects on the flow and heat transfer rates in the cavity. Sivasankaran and Ho (2008) studied numerically the effects of temperature dependent properties of the natural convection of water in a cavity under the influence of a magnetic field. They showed that the heat transfer rate was influenced by the direction of the external magnetic field and was decreased with an increase of the magnetic field. Oztop and Abu-Nada (2008) studied the effects of a partial heater on natural convection using different types and concentrations of nanoparticles. They found that heat transfer was strongly related to types and volume fractions of nanoparticles. Abu-Nada (2009); Abu-Nada (2010) and Abu-Nada et al. (2010) studied the effect of the variables properties of nanofluids in natural convection. They related the deterioration in heat transfer of nanofluids in natural convection to the temperature dependence of nanofluid properties. These findings were also supported by other studies [Abu-Nada and Chamkha (2010a); Abu-Nada and Chamkha (2010b)]. Alam et al. (2012) investigated natural convection in a rectangular enclosure due to partial heating and cooling at vertical walls. Fattahi et al. (2012) applied Lattice Boltzmann Method to investigate the natural convection flows utilizing nanofluids in a square cavity. The fluid in the cavity was a water-based nanofluid containing $\text{Al}_2\text{O}_3$ or Cu nanoparticles. The results indicated that by increasing solid volume fraction, the average Nusselt number increased for both nanofluids. It was found that the effects of solid volume fraction for Cu were stronger than $\text{Al}_2\text{O}_3$. Kefayati et al. (2011) simulated by the Lattice Boltzmann method the natural convection in enclosures using water/$\text{SiO}_2$ nanofluid. The results showed that the average Nusselt number increased with volume fraction for the whole range of Rayleigh numbers and aspect ratios. Also the effect of nanoparticles on heat transfer aug-
mented as the enclosure aspect ratio increased. Lai and Yang (2011) performed mathematical modeling to simulate natural convection of Al$_2$O$_3$/water nanofluids in a vertical square enclosure using the Lattice Boltzmann method. The results indicated that the average Nusselt number increased with the increase of Rayleigh number and particle volume concentration. The average Nusselt number with the use of nanofluid was higher than the use of water under the same Rayleigh number. Mahmoudi et al. (2011) presented a numerical study of natural convection cooling of two heat sources vertically attached to horizontal walls of a cavity. The results indicated that the flow field and temperature distributions inside the cavity were strongly dependent on the Rayleigh numbers and the position of the heat sources. The results also indicated that the Nusselt number was an increasing function of the Rayleigh number, the distance between two heat sources, and distance from the wall and the average Nusselt number increased linearly with the increase in the solid volume fraction of nanoparticles. Kefayati et al. (2013) investigated Prandtl number effect on natural convection MHD in an open cavity which has been filled respectively with liquid gallium, air and water by Lattice Boltzmann Method. They exhibited that heat transfer declines with the increment of Hartmann number, while this reduction is marginal for Ra=10$^3$ by comparison with other Rayleigh numbers. Lattice Boltzmann Method simulation of MHD mixed convection in a lid-driven square cavity with linearly heated wall is investigated by Kefayati et al. (2012). It was demonstrated that the augmentation of Richardson number causes heat transfer to increase, as the heat transfer decreases by the increment of Hartmann number for various Richardson numbers and the directions of the magnetic field. The LBM is an applicable method for simulating fluid flow and heat transfer [Nemati et al. (2010); Mehravaran and Hannani (2011); Pirouz et al. (2011); Mohamad (2007); Succi (2001)]. This method was also applied to simulate the MHD [Martinez et al. (1994)] and, recently, nanofluid [Nemati et al. (2010)] successfully.

The aim of the present study is to assess the ability of Lattice Boltzmann Method (LBM) in solving a nanofluid and a magnetic field simultaneously in the presence of a sinusoidal thermal boundary condition. Moreover, the effects of magnetic field and phase deviations on the heat transfer in the cavity are considered in order to identify the best situation for heat transfer and fluid flow.

2 Mathematical formulation

2.1 Problem statement

A two-dimensional square cavity is considered as shown in Fig. 1. The side walls of the cavity have spatially varying sinusoidal temperature distributions. The horizontal walls are adiabatic. The cavity is filled with water and Al$_2$O$_3$ nanoparticles.
The nanofluid is assumed to be Newtonian and incompressible. The flow is considered to be steady, two dimensional and laminar, while the radiation effects are assumed to be negligible. The thermo-physical properties of the base fluid and the nanoparticles are given in Table 1. The density variation in the nanofluid is approximated by the standard Boussinesq model.

![Diagram](image)

Figure 1: Geometry of the present study with boundary conditions.

<table>
<thead>
<tr>
<th></th>
<th>ρ (kg/m³)</th>
<th>Cₚ (J/kg K)</th>
<th>K (W/mK)</th>
<th>β (K⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>21x10⁻⁵</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>3970</td>
<td>765</td>
<td>40</td>
<td>0.85x10⁻⁵</td>
</tr>
<tr>
<td>Cu</td>
<td>8933</td>
<td>385</td>
<td>400</td>
<td>1.67x10⁻⁵</td>
</tr>
<tr>
<td>TiO₂</td>
<td>4250</td>
<td>686.2</td>
<td>8.9538</td>
<td>0.9x10⁻⁵</td>
</tr>
</tbody>
</table>

Table 1: Thermo-physical properties of water and nanoparticles. [Ghasemi and Aminossadati (2010)]
The magnetic field (strength $B_0$) is applied in the horizontal direction. It is assumed that the induced magnetic field produced by the motion of an electrically conducting fluid is negligible compared to the applied magnetic field. Furthermore, it is assumed that the viscous dissipation and Joule heating can be neglected. Therefore, governing equations are written in dimensional form as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\rho_{nf}(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) = - \frac{\partial p}{\partial x} + \mu_{nf}(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}) \tag{2}
\]

\[
\rho_{nf}(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}) = - \frac{\partial p}{\partial y} + \mu_{nf}(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}) + F_y \tag{3}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf}(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) \tag{4}
\]

Where $F_y$ is the total body forces at $y$ direction and it is defined as follows:

\[
F_y = - \frac{Ha^2 \mu_{nf} v}{H^2} + (\rho \beta)_{nf} g(T - T_m) \tag{5}
\]

Where $Ha$ is $Ha = HB_0 \sqrt{\frac{\sigma_{nf}}{\mu_{nf}}}$

The classical models reported in the literature are used to determine the properties of the nanofluid [Xuan and Roetzel (2000)]:

\[
\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_p \tag{6}
\]

\[
(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_p \tag{7}
\]

\[
(\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_p \tag{8}
\]

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}} \tag{9}
\]

In the above equations, $\phi$ is the solid volume fraction, $\rho$ is the density, $\sigma$ is the electrical conductivity, $\alpha$ is the thermal diffusivity, $c_p$ is the specific heat at constant pressure and $\beta$ is the thermal expansion coefficient of the nanofluid, $\gamma$ is the phase deviation. The effective dynamic viscosity and thermal conductivity of the nanofluid can be modelled by Brinkman (1958); Maxwell (1873):

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \tag{10}
\]
The governing equations are subject to the following boundary conditions:

Bottom wall \( u = v = 0, \frac{\partial T}{\partial y} \bigg|_{y=0} = 0 \)

Top wall \( u = v = 0, \frac{\partial T}{\partial y} \bigg|_{y=H} = 0 \)

Left wall \( u = v = 0, T(0,y) = T_C + A_l \sin(2\pi y/H) \)

Right wall \( u = v = 0, T(H,y) = T_C + A_r \sin(2\pi y/H + \gamma) \)

2.2 Simulation of MHD and nanofluid with Lattice Boltzmann Method

For the incompressible non isothermal problems, the Lattice Boltzmann Method (LBM) is based on two distribution functions, \( f \) and \( g \), for the flow and temperature fields respectively.

For the flow field:

\[
f_i(x + c_i \Delta t, t + \Delta t) = f_i(x, t) - \frac{1}{\tau_v} \left( f_i(x, t) - f_i^{eq}(x, t) \right) + \Delta t F_i
\]

For the temperature field:

\[
g_i(x + c_i \Delta t, t + \Delta t) = g_i(x, t) - \frac{1}{\tau_{\alpha}} \left( g_i(x, t) - g_i^{eq}(x, t) \right)
\]

Where the discrete particle velocity vectors defined by \( c_i \), \( \Delta t \) denotes lattice time step which is set to unity. \( \tau_v, \tau_{\alpha} \) are the relaxation time for the flow and temperature fields, respectively. \( f_i^{eq}, g_i^{eq} \) are the local equilibrium distribution functions that have an appropriately prescribed functional dependence on the local hydrodynamic properties which are calculated with Eqs. (15) and (16) for flow and temperature fields respectively.

\[
f^{eq}_i = \omega_i \rho \left[ 1 + \frac{3 (c_i.u)}{c^2} + \frac{9 (c_i.u)^2}{2c^4} - \frac{3u^2}{2c^2} \right]
\]

\[
g^{eq}_i = \omega_i' T \left[ 1 + \frac{3c_i.u}{c^2} \right]
\]

\( \mathbf{u} \) and \( \rho \) are the macroscopic velocity and density, respectively. \( c \) is the lattice speed which is equal to \( \Delta x/\Delta t \) where \( \Delta x \) is the lattice space similar to the lattice time step \( \Delta t \) which is equal to unity, \( \omega_i \) is the weighting factor for flow, \( \omega'_i \) is the weighting
factor for temperature. D2Q9 model for flow and D2Q4 model for temperature are used in this work so that the weighting factors and the discrete particle velocity vectors are different for these two models and they are calculated with Eqs (17-19) as follows:

**For D2Q9**

\[ \omega_0 = \frac{4}{9} \text{, } \omega_i = \frac{1}{9} \text{ for } i = 1, 2, 3, 4 \text{ and } \omega_i = \frac{1}{36} \text{ for } i = 5, 6, 7, 8 \]  
\( i = 0 \)

\[ c_i = \begin{cases} 0 & i = 0 \\ (\cos \cos[(i - 1) \pi / 2], \sin[(i - 1) \pi / 2])c & i = 1, 2, 3, 4 \\ \sqrt{2} (\cos[(i - 5) \pi / 2 + \pi / 4], \sin[(i - 5) \pi / 2 + \pi / 4])c & i = 5, 6, 7, 8 \end{cases} \]  
(18)

**For D2Q4**

The temperature weighting factor for each direction is equal to \( \omega'_i = 1/4 \).  

\[ c_i = (\cos[(i - 1) \pi / 2], \sin[(i - 1) \pi / 2])c \quad i = 1, 2, 3, 4 \]  
(19)

The kinematic viscosity \( \nu \) and the thermal diffusivity \( \alpha \) are then related to the relaxation time by Eq. (20):

\[ \nu = \left[ \tau_\nu - \frac{1}{2} \right] c_s^2 \Delta t \quad \alpha = \left[ \tau_\alpha - \frac{1}{2} \right] c_s^2 \Delta t \]  
(20)

Where \( c_s \) is the lattice speed of sound which is equals to \( c_s = c / \sqrt{3} \). In the simulation of natural convection, the external force term \( F \) appearing in Eq. (14) is given by Eq. (21)

\[ F_i = \frac{\omega_i}{c_s^2} F. c_i \]  
(21)

Where \( F = F_y \)

The macroscopic quantities, \( u \) and \( T \) can be calculated by the mentioned variables, with Eq. (22-24).

\[ \rho = \sum_i f_i \]  
(22)

\[ \rho u = \sum_i f_i c_i \]  
(23)

\[ T = \sum_i g_i \]  
(24)
2.3 Boundary conditions

The implementation of boundary conditions is very important for the simulation. The distribution functions out of the domain are known from the streaming process. The unknown distribution functions are those toward the domain.

2.3.1 Flow

Bounce-back boundary conditions were applied on all solid boundaries, which mean that incoming boundary populations are equal to out-going populations after the collision.

2.3.2 Temperature

The bounce back boundary condition is used on the adiabatic wall. Temperature at the left and the right walls are known. Since we are using D2Q4, the unknown internal energy distribution functions are evaluated as:

Right wall: \( g_3 = T(y) - g_1 - g_2 - g_4 \) \hspace{1cm} (25)

Left wall: \( g_1 = T(y) - g_2 - g_3 - g_4 \) \hspace{1cm} (26)

2.4 Non-dimensional parameters

By fixing Rayleigh number, Prandtl number and Mach number, the viscosity and thermal diffusivity are calculated from the definition of these non dimensional parameters.

\[ \nu_f = N.Ma.c_s \sqrt{\frac{Pr}{Ra}} \] \hspace{1cm} (27)

Where \( N \) is number of lattices in y-direction. Rayleigh and Prandtl numbers are defined as \( \text{Ra} = g\beta_f H^3 (T_h - T_c) / \nu_f \alpha_f \) and \( \text{Pr} = \nu_f / \alpha_f \) respectively. Mach number should be less than \( Ma = 0.3 \) to insure an incompressible flow. Therefore, in the present study, Mach number was fixed at \( Ma = 0.1 \). The Hartmann number has a very important role for the control of the effect of the magnetic field \( Ha = HB_0 \sqrt{\frac{\sigma_{nf}}{\mu_{nf}}} \). Nusselt number is one of the most important dimensionless parameters in the description of the convective heat transport. The local Nusselt number (Nul and Nur), the average Nusselt number (Nu) and the dimensionless average Nusselt number (Nu*) are calculated as:

\[ \text{Nul} = -\frac{k_{nf}}{k_f} \frac{H}{T_h - T_c} \left. \frac{\partial T}{\partial x} \right|_{x=0} \] \hspace{1cm} (28)
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\[ \text{Nur} = -\frac{k_{nf}}{k_f} \frac{H}{T_h - T_c} \frac{\partial T}{\partial x} \bigg|_{x=H} \]  

(29)

\[ \text{Nu} = \frac{1}{H} \int_{\text{heating half}} \text{Nur} \, dy + \frac{1}{H} \int_{\text{heating half}} \text{Nul} \, dy \]  

(30)

\[ \text{Nu}^*(\phi) = \frac{\text{Nu}(\phi)}{\text{Nu}(\phi = 0)} \]  

(31)

3 Grid testing and validation code:

3.1 Grid testing:

A Lattice Boltzmann Method scheme was used for the numerical simulations. Fig. 2 shows the effect of grid resolution and lattice sizes (20x20), (40x40), (60x60), (80x80) and (100x100) for \( Ha=0 \) and \( \phi = 0 \). By calculating the average Nusselt number for \( Ra=10^3 \) and \( 10^5 \), it was found that a grid size of (100x100) ensures a grid independent solution.

![Figure 2: Average Nusselt number for different uniform grids (\( \phi = 0 \), \( \gamma = \pi/2 \) and \( Ha=0 \)).](image)

3.2 Validation code:

In order to check on the accuracy of the numerical technique employed for the solution of the considered problem, the present numerical code was validated by comparison with the study by Deng and Chang (2008) for the same cavity with sinusoidal boundary conditions for $\gamma = \pi/2$, $Ra=10^5$ and $Pr=0.7$. The results are presented in Fig.3. The results of another validation study comparison with Ghasemi et al. (2011) are presented in Fig. 4 which shows the dimensionless temperature
Figure 4: Comparison of the temperature on axial midline between the present results and numerical results by Ghassemi et al., (2011) ($\phi = 0.03, \text{Ra}=10^5$).

Figure 5: Comparison of the temperature on axial midline between the present results and numerical results by Khanafer et al., (2003) and jahanshahi et al., (2010) ($\text{Pr} = 6.2, \phi = 0.1, \text{Gr}=10^4$).
along the horizontal axial midline of the enclosure for three values of the Hartmann number, for $Ra=10^5$ and for a solid volume fraction $\phi = 0.03$ (excellent agreement is also found). The present code was also validated with the results of Khanafer et al. (2003) and Jahanshahi et al. (2010) for natural convection in an enclosure filled with water/Cu nanofluid for $Ra=6.2\times10^5$ and $\phi=0.1$ as shown in Fig.5.

4 Results and discussion:

Fig. 6 and 7 illustrate the effect of Hartmann number for different values of the Rayleigh number ($Ra = 10^3, 10^4, 10^5$ and $10^6$) and for $\gamma = \pi/2$ on the isotherms and streamlines of nanofluid ($\phi=0.04$) and pure fluid ($\phi=0$). For all Rayleigh number it demonstrates that the effect of nanoparticles on the isotherms decreases with the augmentation of Hartmann number. The thickness of the boundary layer decreases with the rise of Rayleigh number, the opposite effect occurs with the increase of the Hartmann number. Fig.8.a shows the variation of average Nusselt number as function of Hartmann number for different Rayleigh number, the increase of Rayleigh number increases the heat transfer rate, on the contrary, the increase of the Hartmann decreases the heat transfer rate. The streamlines shows that the flow behavior is affected with the change in the Rayleigh number and the Hartmann number. At $Ra = 10^3-10^4$ and in the absence of magnetic field, the flow is characterized by two cells, one above the other, rotating in opposite direction inside the enclosure. The minor cell near the let-top corner is elongated when the Hartmann number is increased to 30 and 60 and when Rayleigh number is increased to $10^5$ and $10^6$ also a third cell appears near the right-bottom corner. The strength of these cells increases as the Rayleigh number increases and decreases as the Hartmann number increases. For all values of Rayleigh number, the application of the magnetic field has the tendency to slow down the movement of the fluid in the enclosure. The braking effect of the magnetic field is observed from the maximum intensity of circulation $|\psi|_{max}$ (Fig.8.b) presents the variation of the maximum value of the stream function as a function of Hartman number for several values of Rayleigh number for $\phi=0$ and $\gamma = \pi/2$. It is observed that the effect of Hartmann number is opposite to the effect of Rayleigh number. For $Ra = 10^3$ and $10^4$, $|\psi|_{max}$ is constant and small for all values of Hartmann number. The conduction is dominant. For $Ra = 10^5$ and $10^6$, the convection is dominant for low values of Hartmann number, more than the Hartmann number increases convection is more disadvantaged, until reaching the conductive regime.

Fig.9. a and b illustrate the variations of the local Nusselt numbers along the left sidewall and right sidewall at various Rayleigh numbers for $Ha=0$ and 60. For both walls, the curves drawn for the Nusselt numbers against $y/H$ are approximately of sinusoidal shape like the thermal boundary. This indicates that the local heat
Figure 6: isotherms for different Hartmann and Rayleigh numbers and for $\gamma = \pi/2$. 

(—) $\phi = 0.04$ and (- - -) $\phi = 0$. 

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Figure 7: Streamlines for different Hartmann and Rayleigh numbers and for $\gamma = \pi/2$, (—) $\phi = 0.04$ and (— —) $\phi = 0$. 
Figure 8: Variation of the maximum of the average Nusselt number (a) and stream function (b) with Hartmann number for different Rayleigh number for $\gamma = \pi/2$ and $\phi = 0$. 
transfer is directly affected by the temperature distribution on the surface. In other words, larger heat transfer occurs when the temperature is higher. In the left sidewall, it is obviously understood that the lower half \((0 \leq y/H \leq 0.5)\) is the heating half and the upper half \((0.5 \leq y/H \leq 1)\) is the cooling half. The variations of the local Nusselt numbers along the left sidewall and the right sidewall are exhibited in Fig.10.a and b for various Hartmann numbers. At \(Ra \leq 10^4\) the heat transfer gets no remarkable change on both sidewalls even if the Hartmann number is increased but for \(10^4 < Ra \leq 10^6\) it seems that the Nusselt number decreases while the Hartmann number is increased.

(a)

(b)

Figure 9: Variation of the local Nusselt number on the left and the right walls for different Rayleigh number for \(Ha=0\) (a) and \(Ha=60\) (b) for \(\gamma = \pi/2\) and \(\phi = 0\).
Figure 10: Variation of the local Nusselt number on the left and the right walls for different Hartmann number for $Ra=10^4$ (a) and $Ra=10^6$ (b) for $\gamma = \pi/2$ and $\phi = 0$.

Fig.11 shows comparison of the average Nusselt number and the dimensionless average Nusselt number for various Hartmann and Rayleigh numbers at different volume fractions for $\gamma = \pi/2$. The average Nusselt number demonstrates that heat transfer increases with the enhancement of Hartmann number at $Ra=10^3$. For $10^4$ heat transfer declines with the enhancement of Hartmann number from Ha=0 to 30 but the average Nusselt number of Ha=90 is more than Ha=60. Indisputably, the best parameter for showing the effect of the addition of nanoparticles to the
Figure 11: Variation of the average Nusselt number and dimensionless average Nusselt number as function of solid volume fraction for different Hartmann number for $\gamma = \pi/2$, $Ra=10^3$ (a) $Ra=10^4$ (b) and $Ra=10^5$ (c).
Figure 12: Variation of the local Nusselt number on the left and the right walls for different solid volume fraction at $\gamma = \pi/2$, $Ra=10^5$ for $Ha=0$ (a) and $Ha=90$ (b).

pure fluid is the dimensionless average Nusselt number. At $Ra=10^3$, the best effect of nanoparticles is obtained for $Ha = 0$, by increasing the Hartmann number the effect of nanoparticles decreases. At $Ra=10^4$ the lowest effect of nanoparticles is obtained for $Ha = 0$, for $Ha=30-90$ the nanoparticles have the same tendency to the increase of the solid volume fraction. At $Ra=10^4$ the lowest effect of nanoparticles is obtained for $Ha = 0$, for $Ha=30-90$ the nanoparticles have the same tendency to the increase of the heat transfer. At $Ra=10^5$, the augmentation of Hartmann number play a positive role in the improvement of nanoparticles effect on heat transfer albeit
Figure 13: Isotherms for different Rayleigh number and phase deviations for $Ha=0$, $(- - -) \phi = 0.04$ and $( - - ) \phi = 0$. 
Figure 14: Streamlines for different Rayleigh number and phase deviations for \( \text{Ha}=0 \). (—-\) \( \phi = 0.04 \) and (- - -) \( \phi = 0 \).
Figure 15: Variation of the local Nusselt number on the left and the right walls for different phase deviations at $Ha = 0$ and $\phi = 0$ for $Ra=10^3$ (a) and $Ra=10^5$ (b).

the tendency ceases from $Ha=60$ to 90.

Fig.12.a and b indicate the local Nusselt number on the right and left sidewalls for various volume fractions at $Ra=10^5$, $\gamma = \pi/2$ and $Ha=0$-90. It is shown that the effect of nanoparticles is more significant for $Ha = 90$ which is consistent with Fig.11.

Fig.13 and 14 illustrate the effect of Rayleigh number ($Ra=10^3, 10^4$ and $10^5$) for different phase deviation ($\gamma = 0, \pi/4, 3\pi/2$ and $\pi$) and for $Ha=0$ on the isotherms
Figure 16: Variation of the average Nusselt number and dimensionless average Nusselt number as function of solid volume fraction for different phase deviations for $Ha=0$, $Ra=10^3$ (a) $Ra=10^4$ (b) and $Ra=10^5$ (c).
and streamlines of nanofluid ($\phi=0.04$) and pure fluid ($\phi=0$). It shown the isotherms along the left sidewall are retained. Hence, the heat transfer on the left sidewall is kept fixed, but that on the right sidewall is varied. At $\gamma = 0$, for $Ra < 10^5$ four cells are formed with approximately symmetries about middle of the cavity, for $Ra=10^5$ symmetry is broken only for $\phi=0$. As the phase deviation increases up to $\gamma = \pi/4$, a multi-cellular flow structure is formed in the cavity with one large diagonal cell and two smaller corner cells. As the phase deviation increases, the size of the upper left-corner cell is enlarged but the lower right-corner cell disappears. At $\gamma = \pi$, the flow structure is of two identical cells in the enclosure.

Fig. 15. a and b show the effect of the phase deviation for $Ra=10^3$ and $10^5$ on the local Nusselt number along the y coordinates of the two vertical sidewalls at $\phi=0$ and Ha=0. At $Ra=10^3$ it is observed that the heat transfer of the left wall is not affected so much on changing the phase deviation, but the heat transfer of the right wall is affected significantly on changing the phase deviation from $\gamma = 0$ to $\pi$. The local Nusselt number curves are approximately of sinusoidal shape like the thermal boundary along the vertical walls. This clearly shows that the local heat transfer is directly affected by the temperature distribution on the surface. It is also found that a higher heat transfer occurs where the temperature is higher. At $Ra=10^5$ the local Nusselt number along the right side wall is greatly affected by changing the phase deviation. It is also found that the local Nusselt number is increased as the
Rayleigh number increases.

Fig. 16 shows the effects of volume fractions and phase deviations for various Rayleigh numbers on the average Nusselt number and the dimensionless average Nusselt number. For all Rayleigh number and phase deviations the heat transfer increases with the rise of volume fraction. For $Ra=10^3$, heat transfer decreases from $\gamma=0$ to $\pi/4$ and increases from $\gamma=\pi/2$ to $\pi$. Moreover, the dimensionless average Nusselt number has the same trend in different phase deviations. The nanofluids have effects very similar for all phase deviations. At $Ra=10^4$ and $10^5$, heat transfer increases with the rise of phase deviations, the most heat transfer was obtained in $\gamma=\pi$. The best effect of nanoparticles for $Ra=10^4$ and $10^5$ is obtained in $\gamma=0$ and $\pi/4$ respectively.

Fig. 17 shows the effect of nature of nanoparticles on heat transfer. Three nanoparticles are compared at $Ra=10^5$, $Ha=0$ and $\gamma=\pi/2$. The heat transfer depends strongly on the nano thermal conductivity, so water-Cu nanofluid enhances the heat transfer compared with water-Al$_2$O$_3$ and water-TiO$_2$. Table 1 shows the proportionally to the solid volume fraction.

5 Conclusions:

In this paper the effect of a magnetic field on a nanofluid flow in a cavity with a sinusoidal thermal boundary condition has been analyzed in the framework of a Lattice Boltzmann Method. The main conclusions can be summarized as follows:

- The good agreement with earlier numerical results demonstrates that the Lattice Boltzmann Method is an appropriate technique for these problems.

- For $\gamma=\pi/2$, heat transfer and fluid flow decrease with an increase in the Hartmann number while they increase with an increase in the Rayleigh number.

- At $\gamma=\pi/2$, the growth of nanoparticles volume fraction improves heat transfer for Hartmann number from $Ha=0$ to $90$ and for Rayleigh number from $Ra=10^3$ to $10^5$. For $Ra=10^5$, the most evident effect of nanoparticles is obtained for $Ha=90$.

- For all phase deviations the growth of nanoparticles volume fraction improves heat transfer. At $Ra=10^5$ and $Ha=0$, the heat transfer rate increases with the rise of phase deviations, the most evident effect of nanoparticles is obtained for $\gamma=\pi/4$. 
References


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