The functional dimensioning: application of vector algebra to the design of mechanical parts. study case.

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Summary
The sizing of the component parts of a mechanical assembly must first assure its assembly and functionality and then the replacement (interchangeability) of used parts with new parts. For diametral fits (gaps and interferences) the ISO system of limits and fits is used, but for axial gaps the functional dimensioning method is applied to ensure maximum and minimum functional values. For each axial gap, the axial dimensions of the involved parts must be identified. Later, a minimal chain of dimensions is established, defined as the addition and subtraction of vectors parallel to the axis of interest. The difference between the maximum and minimum allowable axial gap is its dimensional tolerance that will be distributed among all components of the dimensional chain. The resultant dimensional tolerance for each dimension should be obtainable through the normal machining processes. In the mechanical design of a CO₂ laser were identified 18 functional axial gaps, and to determine the tolerance of each dimension were considered ISO Tolerance Grades IT9 to IT13, but rounded to 0.05 mm in order to verify them with a Vernier caliper with reading error of 0.05 mm.

Keywords: Functional dimensioning, interchangeability, vector algebra, axial gaps, chain dimensions.

Introduction
For the suitable operation of a mechanical set, diametral fits (radial) and longitudinal plays must be set. The diametral fittings involve only two dimensions: the diameter of a hole and the diameter of the tree that must be inserted in the hole. The ISO Standard 286 of limits and fits is applied for the fit selection. The fit can be with play (fig. 1a) or with interference (fig. 1b). A fit is a functional condition.

The longitudinal gaps can involve two or more pieces. If the gap depends only on two parts, its size can also be selected according to ISO Standard 286 of limits and fits (fig. 2a). If the functional condition depends on three or more parts (fig. 2b), to determine the tolerances of the three or more lengths with which the required maximum and minimum gap values are guaranteed, the method of functional dimensioning must be applied.

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The method of functional dimensioning is an application of vector algebra (fig. 3), where all vectors are parallel to the gap axis. It can be summed as follows:

a. The axial gap is considered like a resulting vector,

b. The axial dimension of each involved part is identified, and is considered as a vector in a summing vector chain,

c. Starting off of the origin of the resulting vector, one begins the chain of dimensions,

d. The last vector of the chain must be placed from the end of the last dimension to the destiny of the resulting vector (the axial gap).

When it is difficult to verify a part dimension in a chain of dimensions, this dimension can be obtained as a resultant of the sum or subtraction of two dimensions that can be verified (fig. 4). In this case it will be known as a transferred dimension.

Method
This project was built after reviewing two previous 25W CO₂ laser beam prototypes [Godoy, Yankelevich, Rodriguez, Dominguez and Herrera (1986); Lomelí-Mejía (1996)] where several design errors were detect just until the experimentation phase. Figure 5 shows part of the frontal head for this design [Damian-Noriega, Lomeli-Mejía, Perez-Moreno, Villanueva-Pruneda and Montes-Estrada
The functional dimensioning

(2008)]. Figure 6 shows the functional condition JB, which assures that the bushing (part 5) tights on both o-rings (part 3) and on the other bushing (part 4) against the frontal head (part 1). For each functional condition its minimum dimensional chain has to be drawn. Figure 7 shows the dimensional chain for the JD condition. This functional condition assures the radial compression of the o-ring on an inner cylindrical surface.

According to vector algebra (figure 7):

\[ JD_{\text{max}} = C_{1\text{max}} - 2A_{3\text{min}} - A_{4\text{min}} - D_{5\text{min}} \]  

(1)

\[ JD_{\text{min}} = C_{1\text{min}} - 2A_{3\text{max}} - A_{4\text{max}} - D_{5\text{max}} \]  

(2)
The standardized degrees for dimensional tolerance that were considered in this work went from IT9 to IT13 [International Organization for Standardization (1986)], corresponding to an average machining work. Values were rounded to 0.05 mm in order to be able to read them with a Vernier caliper, reading error of 0.05 mm.

**Results**

Table 1 contains the dimensions and respective tolerances proposed to satisfy the functional condition JD. D1 is a transferred dimension measured between D1Ta and D1Tb. An IT degree between parentheses indicates that the value of the tolerance has been adjusted near to this degree. Without parenthesis, the value corresponds to IT degree as shown.

After determining the chain of dimensions for all the functional gaps in which the part No 5 is involved, its functional fitting turns out to be the one that is shown in figure 8.
Table 1: Proposed dimensions to satisfy JD.

<table>
<thead>
<tr>
<th>Functional Condition</th>
<th>JD</th>
<th>D1</th>
<th>D5</th>
<th>A3</th>
<th>A4</th>
<th>A3</th>
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</table>

Discussion

This paper presents only a section of the hole functional analysis used to determine all the parts dimensions that makes the front and rear head of a CO2 laser beam prototype based on a compact mechanical design [Damián-Noriega, Lomelí-Mejía, Pérez-Moreno, Villanueva-Pruneda, Montes-Estrada (2008)].

The next step is the parts manufacturing, its assembly and experimental characterization. Only the mirror is an imported part, the rest are commercial homemade parts.

Figure 8: Functional dimensions for part # 5.

This dimensioning method had its origin at the French automotive factory Citroën, almost 60 years ago. In its origins, its use was exclusively at Citroën, but later
it was thought at the big french engineering schools by Professor Jacques Bielle (figure 9) [Bielle (2009)], who can be considered as the father of the functional dimensioning.

Rounding dimensions and assigning symmetrical deviations is a very popular mistake if the functional dimensioning method is not applied, and in this case the outcome could be disastrous. The use of this method avoids a lot of problem in the final assembly.

It could be mentioned two cases in México, one a success and the other a very unfortunate one. The first: a small hydraulic cylinders manufacturing factory had a lot of parts rejection in the assembly stage, which implied a delivery delay. This company faced bankrupt because the cylinders were being imported by its competitors. Professor Villanueva (coauthor of this work) solved this assembly problem applying the functional dimensioning method.

Nowadays, this company associated with a Canadian counterpart is exporting its production.

Second case: a 50 HP steam turbine was designed to be sold to the Mexican petroleum company, PEMEX. After manufacturing part for almost four months, the assembly stage was initiated. Until then the design department found that the parts dimensioning was incorrect, and it was necessary to manufacture some parts in order to be able to reach the assembly. As a result of this, the manufacturing company was punished for the delivery delay of the equipments.

Figure 9: Professor J. Bielle, father of the functional dimensioning.

Acknowledgement

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References


