Simulation of Tidal Current, Wind and Wave for Numerical Navigation of Ship

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Abstract: This paper deals with the numerical estimation methods of the tidal current, the wind and the wave in the coastal sea area. Such estimates are very important because the tidal current, the wind and the wave react to effect manoeuvring of a advanced ship. The numerical estimations of the tidal current, wind and wave were carried out in the bay. As an application of these estimations, numerical navigation of a sailing ship was conducted. It was proved that the effects of the tidal current, the wind and the wave in coastal sea on a sailing ship could be accurately estimated for a navigational simulation.

1 Introduction

Recently, the numerical estimation of the tidal current, the wind and the wave in the coastal sea area give much attention to the scope of a ship navigation. A sailing ship is endangered by strong winds or higher waves produced by low pressure or typhoons passing near the shipping rout. Also, An advanced ship drifts from the planed course by the strong tidal current.

The object of this research is to explain the simulation method of the tidal current, the wind and the wave in the coastal area, and to apply on the numerical navigation. First, numerical computational methods of the tidal current, the wind and the wave were discussed using POM, MM5 and SWAN separately. Second, as the application of the numerical estimation of these weather, the navigational simulations of a sailing ship under the effects of ones were conducted. It was recognized that these estimations of the tidal current, the wind and the wave in the coastal area may be very useful for the purpose of the routing of a sailing ship.

2 Simulation of Tidal Current

The numerical simulation of the tidal current was carried out by using POM, which was developed by Princeton University Oceanographic Center [Mellor (2004)].

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The coordinate of the $z$ axis direction is arranged to the $\sigma$ coordinate system as $\sigma = (z - \eta)/(H + \eta)$ and $D = H + \eta$, where $H$ is the water depth from the mean water surface, $\eta$ is the tidal level. The basic equations of the tidal current are as follows:

$$\frac{\partial DU}{\partial x} + \frac{\partial DV}{\partial y} + \frac{\partial \omega}{\partial \sigma} + \frac{\partial \eta}{\partial t} = 0 \quad (1)$$

$$\frac{\partial UD}{\partial x} + \frac{\partial U^2 D}{\partial x} + \frac{\partial UV D}{\partial y} + \frac{\partial U \omega}{\partial \sigma} - fV D + gD \frac{\partial \eta}{\partial x} + \frac{gD^2}{\rho_0} \int_\sigma^0 \left[ \frac{\partial \rho'}{\partial x} - \frac{\sigma'}{D} \frac{\partial D}{\partial x} \frac{\partial \rho'}{\partial \sigma} \right] d\sigma' = \frac{\partial}{\partial \sigma} \left[ \frac{K_M \partial U'}{D \partial \sigma} \right] + F_x \quad (2)$$

$$\frac{\partial VD}{\partial t} + \frac{\partial UV D}{\partial x} + \frac{\partial V^2 D}{\partial y} + \frac{\partial V \omega}{\partial \sigma} + fUD + gD \frac{\partial \eta}{\partial y} + \frac{gD^2}{\rho_0} \int_\sigma^0 \left[ \frac{\partial \rho'}{\partial y} - \frac{\sigma'}{D} \frac{\partial D}{\partial y} \frac{\partial \rho'}{\partial \sigma} \right] d\sigma' = \frac{\partial}{\partial \sigma} \left[ \frac{K_M \partial v'}{D \partial \sigma} \right] + F_y \quad (3)$$

where $(U, V)$ and $\omega$ are components of tidal current in the horizontal and the normal direction to the $\sigma$ plain, $f$ is the Coriolis coefficient, $g$ is the acceleration of gravity, $(F_x$ and $F_y$) are the horizontal viscosity diffusion coefficients and $K_M$ is the frictional coefficient of the sea bottom.

![Figure 1: Simulated ocean surface wind in Osaka Bay](image)

The numerical simulation of the tidal current in a bay was carried out in Osaka Bay in Japan. The data of the water depth in the bay was obtained from the website of the Japan Oceanographic Data Center, and the geographic data of the bay
was obtained from the National Geographic Data Center in the U.S.A. The grid interval of the x- and y-axes is 0.25 minutes. The tidal harmonic component M2 is implemented in the open boundary. The velocity distribution of the tidal current in Osaka Bay on time of maximum flood and ebb tides is shown in Fig. 1. As a result, the numerical simulation of tidal currents obtained with POM model agreed satisfactorily with the tidal current in Osaka Bay.

3 Simulation of Wind

Numerical simulation of wind was carried out by MM5, which is a meso-scale meteorological model developed by Pennsylvania State University [Jimy and Dave (2005)]. The $\sigma$ coordinate system was applied as $\sigma = (P - P_t)/(P_s - P_t)$, which $P$ is the air pressure at an arbitrary height, $P_s$ is the surface pressure, and $P_t$ is a specified constant top pressure of calculated region. The basic equation of air pressure are as follows:

$$\frac{\partial p'}{\partial t} - \rho_0 gw + \gamma p \nabla \cdot V = -V \cdot \nabla p' + \gamma p \left( \frac{\dot{Q}}{c_p} + \frac{T_0}{\theta_0} D_\theta \right)$$

(4)

where $p'$ is perturbation pressure of $p$, $\rho_0$ is the standard air density, $g$ is the acceleration of gravity, $w$ is velocity component of the $z$ direction, $\gamma$ is the heat rate, $V$ is the horizontal vector of the wind, $T$ is the air temperature of K, $Q$ is the in-adiabatic efficiency of $J/(kg \cdot s)$, $c_p$ is the specific heat at constant pressure, $T_0$ is the standard air temperature, $\theta_0$ is the potential temperature, and $D_\theta$ is the diffusion of $\theta_0$. The horizontal and vertical momentum equations are as follows:

$$\frac{\partial u}{\partial t} + \frac{m}{p} \left( \frac{\partial p'}{\partial x} - \sigma \frac{\partial p'}{\partial x} \frac{\partial p'}{\partial \sigma} \right) = -V \cdot \nabla u + v \left( f + u \frac{\partial m}{\partial y} - v \frac{\partial m}{\partial x} \right) - e w \cos \alpha - \frac{uw}{r_{earth}} + D_u$$

(5)

$$\frac{\partial v}{\partial t} + \frac{m}{p} \left( \frac{\partial p'}{\partial y} - \sigma \frac{\partial p'}{\partial y} \frac{\partial p'}{\partial \sigma} \right) = -V \cdot \nabla v + u \left( f + u \frac{\partial m}{\partial y} - v \frac{\partial m}{\partial x} \right) - e w \sin \alpha - \frac{vw}{r_{earth}} + D_v$$

(6)

$$\frac{\partial w}{\partial t} - \frac{\rho_0}{\rho} \frac{g p'}{\gamma p} + \frac{g p'}{\gamma p} = -V \cdot \nabla w + g \frac{P_0 T'}{P_t} \frac{R_d}{c_p} \frac{p'}{p} + e(u \cos \alpha - v \sin \alpha) + \frac{u^2 + v^2}{r_{earth}} + D_w$$

(7)

where ($u$ and $v$) are the velocity components of $V$, $m$ is the horizontal curvature term, $\rho$ is the air density, $p^*$ is $P_s - P_t$, $f$ is the coefficient of Coriolis, $\alpha$ is the
rotation angle of the grid and the northern axis of the grid, \((D_u \text{ and } D_v)\) are the components of the coefficients of the horizontal diffusion, and \(\text{r}_{\text{earth}}\) is the vertical curvature term. \(p_0\) is the standard air pressure, \(T'\) is the perturbation temperature of \(K\), \(R_d\) is the constant value of dry air, and \(D_w\) is the vertical diffusion.

![Figure 2: Simulated ocean surface wind in Osaka Bay](image)

The numerical calculation of the wind was conducted at Osaka Bay. The simulation of the wind lasted 36 hours, during which time the fully developed low pressure passed over the Nippon Sea, located in the north part of Osaka. Before the low pressure passed, a strong southern wind blew, and, after that, a northern wind blew as shown in Fig. 2. As a result, the numerical simulation agrees satisfactorily with the observed ones.

### 4 Simulation of Wave

Numerical simulation of the wave was carried out by SWAN model, which was developed by at Delft University of Technology [The SWAN team (2009)]. In SWAN model, the wave is expressed by the action density \(N(\vec{x},t,\sigma,\theta)\) in space \(\vec{x}\) and time \(t\). The action density is defined as \(N = E/\sigma\), which \(E\) is the directional spectrum. The action density \(N\) is governed by the action balance equation as follow,

\[
\frac{\partial N}{\partial t} + \nabla_{\vec{x}} \cdot [(\vec{c}_g + \vec{U})N] + \frac{\partial c_\sigma N}{\partial \sigma} + \frac{\partial c_\theta N}{\partial \theta} = \frac{S_{\text{tot}}}{\sigma}
\]  

The first term of the left-hand side is the time variation of \(N\), the second term denotes the propagation of wave energy in two-dimensional geographical \(\vec{x}\)–space, \(\vec{c}_g\) is the group velocity of wave which is expressed by \(\vec{c}_g = \partial \sigma / \partial k\) following
the dispersion relation $\sigma^2 = g|\vec{k}| \tanh(|\vec{k}|d)$, $\vec{k}$ is the wave number vector and $d$ is the water depth. The third term represents the effect of shifting of the radian frequency due to variation in depth and mean currents. The fourth term represents depth-induced and current-induced refraction. The quantities $c_\sigma$ and $c_\theta$ are the propagation velocities in spectral space $(\sigma, \theta)$. The right-hand side contains $S_{\text{tot}}$, which is the source/sink term that represents all physical processes which generate, dissipate, or redistribute wave energy.

Fig. 3 shows the numerical calculated fully development wave generated by the wind of the southern direction in shown in Fig. 2(a). The wave gradually developed down the wind. The distributions of the higher wave height distributed in the interior bay. Numerical results of waves were agreed satisfactory with the observed data.

![Figure 3: Simulated wave (Wind : direction=189 degrees, velocity=11 m/s)](image)

5 Numerical Navigation in Tidal Current, Wind and Wave

Simulation of numerical navigation was carried out to examine the effect of the tidal current, the wind and the wave on a running ship. The model ship numerically sailed with the velocity of 12.3 knots on the virtual course sails in the tidal current, the wind and the wave in Osaka Bay. The ship position at each time was estimated by MMG of the ship manoeuvring theory. The course lines drifted by the tidal current, the wind and the wave were compared with the dead-reckoning track without ones. It was proved that the effects of these weather in the bay on a sailing ship could be accurately estimated in the simulation.
6 Conclusion

The numerical estimations of the tidal current, the wind and the waves in the bay were carried out by using the POM, MM5 and SWAN. These simulated results were applied to the numerical navigation. The numerical estimation methods of these weather provided satisfactory results for numerical navigation.

References

