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Role of Coupling Terms in Constitutive Relationships of Magnetostrictive Materials

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Abstract: Anhysteretic, coupled, linear and nonlinear constitutive relationship for magnetostrictive material is studied in this paper. Constitutive relationships of magnetostrictive material are represented through two equations, one for actuation and other for sensing, both of which are coupled through magneto-mechanical coefficient. Coupled model is studied without assuming any explicit direct relationship with magnetic field. In linear-coupled model, which is assumed to preserve the magnetic flux line continuity, the elastic modulus, the permeability and magneto-elastic constant are assumed as constant. In nonlinear-coupled model, the nonlinearity is decoupled and solved separately for the magnetic domain and mechanical domain using two nonlinear curves, namely the stress vs. strain curve and magnetic flux density vs. magnetic field curve. This is done by two different methods. In the first, the magnetic flux density is computed iteratively, while in the second, artificial neural network is used, where in the trained network will give the necessary strain and magnetic flux density for a given magnetic field and stress level. The effect of nonlinearity is demonstrated on a magnetostrictive rod.

keyword: Magnetostrictive Material, Magnetomechanical Coupling, Artificial Neural Network.

1 Introduction

Some magnetic materials (magnetostrictive) show elongation and contraction in the magnetization direction due to an induced magnetic field. This is called the magnetostriction, which is due to the switching of a large amount of magnetic domains caused by spontaneous magnetization, below the Curie point of temperature. Thus magnetostrictive materials have the ability to convert magnetic energy into mechanical energy and vice versa. This coupling between magnetic and mechanical energies represents the transudation capability that allows a magnetostrictive material to be used in both actuation and sensing devices. Due to magnetostriction and its inverse effect (also called Villery effect)[Villery (1865)], magnetostrictive materials can be used both as an actuator and as well as a sensor.

The theoretical and experimental study of magnetostrictive materials has been the focus of considerable research for many years. However, only with the recent development of giant magnetostrictive materials (e.g. Terfenol-D), it is now possible to produce sufficiently large strains and forces to facilitate the use of these materials in actuators and sensors. This has led to the application of magnetostrictive materials to such devices as micro-positioners, vibration controller, sonar projectors and insulators, etc. Magnetostrictive material has found its way in many structural application such as vibration control, noise control and structural health monitoring.

The use of this material in smart laminated composites for vibration suppression, is examined by many researchers. Reddy and Barbosa (2000) investigated laminated composite beams containing magnetostrictive layers modelled as distributed parameter systems to control the vibration suppression. The effect of material properties, lamination scheme, and placement of the magnetostrictive layers on vibration suppression were investigated. Pelinescu and Balachandran (2001) presented analytical investigations conducted into active control of longitudinal and flexural vibrations transmitted through a cylindrical strut fitted with piezoelectric and magnetostrictive actuators. RoyMahapatra, Gopalakrishnan, and Balachandran (2001) have used this material to suppress all the frequency gear box noise components for active noise control in helicopter passenger cabin. Anjanappa and Bi (1994) have developed an integrated model to analyze the vibration suppression capability of a cantilever beam embedded with magnetostrictive

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mini actuator using the Euler-Bernoulli beam theory and strain energy conservation principle. Saidha, Naik, and Gopalakrishnan (2003) experimentally demonstrated the use of this material for structural health monitoring of composite beams.

One of the main issues in the design of these magnetostrictive sensors/actuators is to predict its behavior under various mechanical and/or magnetic excitation conditions through the constitutive relationship of material. Constitutive relationship of magnetostrictive materials consists of a sensing and an actuation equation. In sensing equation magnetic flux density is function of applied magnetic field and stress where as, in actuation equation, strain is function of applied magnetic field and stress. Both sensing and actuation equations are coupled through applied magnetic field and mechanical stress level.

Analysis of smart structures using magnetostrictive materials are generally performed using uncoupled models. Uncoupled models are based on the assumption that the magnetic field within the magnetostrictive material is proportional to the electric coil current times the number of coil turn per unit length [Ghosh and Gopalakrishnan (2004)]. Due to this assumption, actuation and sensing equations gets uncoupled. where, for actuator, the strain due to magnetic field (which is proportional to coil current) is incorporated as the equivalent nodal load in the finite element model for calculating the block force. Thus, with this procedure, analysis is carried out without taking smart degrees of freedom in the finite element model. Similarly for sensor, where generally coil current is assumed zero, the magnetic flux density is proportional to mechanical stress, which can be calculated from the finite element results through post-processing. This assumption on magnetic field, leads to the violation of flux line continuity, which is one of the four Maxwell's equations in electromagnetism.

On the other hand, in coupled model, it is considered that magnetic flux density and/or strain of the material are functions of stress and magnetic field, without any additional assumption on magnetic field, like uncoupled model. Benbouzid et al. modelled the static [Benbouzid, Reyne, and Meunier (1993)] and dynamic [Benbouzid, Kvarnsjo, and Engdahl (1995)] behavior of the nonlinear magneto-elastic medium for magneto-static case using finite element method. Magneto-mechanical coupling was incorporated considering both permeability and elastic modulus as functions of stress and magnetic field. However, all these work do not provide a convenient way for analysis of magnetostrictive smart structure considering coupled magneto-mechanical features. This paper deals with the constitutive relationship considering coupled features of magnetostrictive materials, which can be used in a finite element formulation considering both magnetic and mechanical degrees as unknown degrees of freedom. In addition, it is shown that the magnetic field is not proportional of applied coil current (which is the assumption of uncouple model), and it depends on the mechanical stress on the magnetostrictive material. This paper also shows that coupled model preserve the flux line continuity, which is one of the drawback of uncoupled model.

The constitutive relations of magnetostrictive materials are essentially nonlinear [Butler (1988)]. The prediction of behavior of magnetostrictive material, in general, is extremely complicated due to its hysteretic non-linear character. In structural application, due to this nonlinear material properties, modelling of the system will become nonlinear, for which exact non-linear constitutive relationships is essential. Toupin (1956) and Maugin (1985) had done extensive work related to electrostrictive and piezoelectric phenomena which have similarities in form with the magnetostriction phenomena. Earlier study to model uncoupled nonlinear actuation of magnetostrictive material was done by KrishnaMurty, Anjanappa, Wang, and Chen (1999) by considering a fourth order polynomial of magnetic field for each stress level. In this approach, stress level for which curve is not available, the coefficients of the curve will have to be interpolated from the coefficients of nearest upper and lower stress level curves.

In this paper, Nonlinear constitutive relationship of this material is studied considering coupled model, where nonlinear strain and nonlinear magnetic flux density response from magnetic field and stress is studied considering both sensing and actuation equation simultaneously. To reduce the complexity of nonlinearity in magnetostrictive material, both mechanical and magnetic nonlinearity are considered separately. In this approach only two nonlinear curves, one for mechanical stress-strain and other for magnetic field-flux density relationships are essential to model this nonlinearity in their respective domains. This paper has shown perfect decoupling of nonlinearity by rewriting sensing and actuation

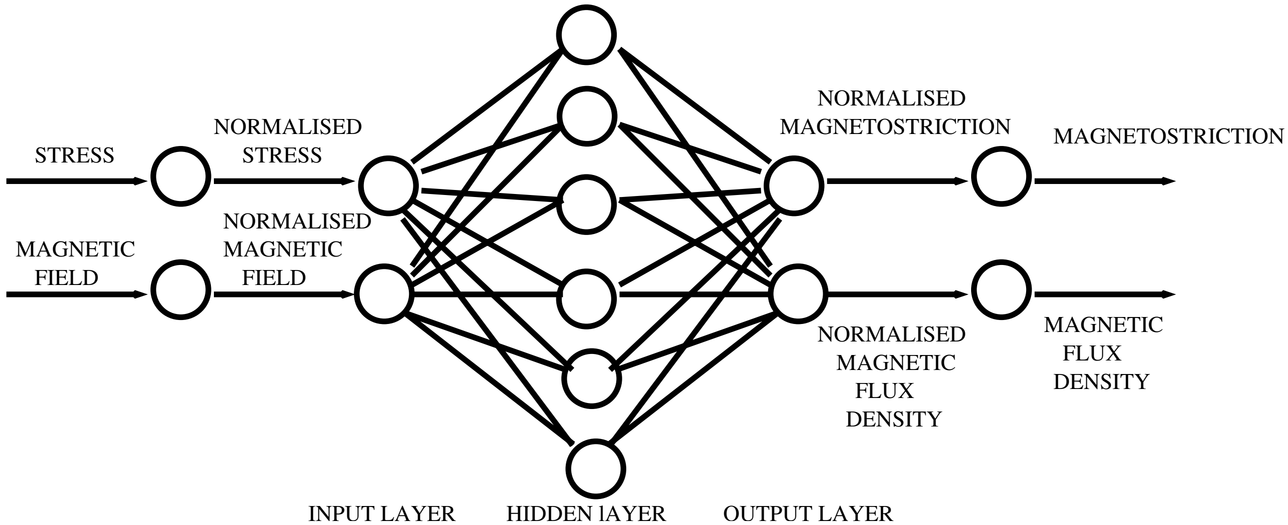

Figure 1 : Artificial neural network architecture

equation in terms of magnetic flux density and strain instead off magnetic field and strain.

In this model magnetic flux density and strains are computed from magnetic field and stress level through an iterative procedure due to these nonlinear curves. To avoid this nonlinear iteration, one three layer artificial neural network (ANN) is trained to get this nonlinear mapping directly. ANN is an universal approximater, which can give a nonlinear parameterize mapping from a given input data to an output data. In this paper it is shown that ANN can be used to get the direct mapping for constitutive relationship of magnetostrictive materials, where inputs in the network are magnetic field and applied stress level and outputs in the network are the strain and the magnetic flux density. Hence, nonlinearity in elastic modulus and permeability is replaced by this trained network.

This paper is organized as follows. First the artificial neural network as an universal function approximatior is introduced. Then the anhysteretic modelling of magnetostrictive material for linear-coupled and nonlinear-coupled constitutive relationships is given. In both linear and nonlinear models total mechanical and magnetic energy is calculated for a magnetostrictive rod. Hamiltonian principle is used to get the equations for magnetic and mechanical degrees of freedoms. In linear model, modulus of elasticity, permeability and magneto-mechanical coefficient of magnetostrictive rod are considered as constant. In nonlinear model, nonlinear stress-strain and magnetic flux density-magnetic field relationships is assumed to predict highly nonlinear behavior of magnetostrictive materials. Even in this case, the magneto-mechanical coefficient is taken as constant. Using this model the strain and magnetic flux density is computed from applied stress level and coil current through iterative procedure. As the nonlinear-coupled model requires iteration to get magnetic flux density and strain from applied stress and coil current, one artificial neural network is trained to avoid the iteration or for at least to get a initial guess for the iteration. Next the ANN model is discussed. One three layer with 4 nodded hidden layer ANN is trained using some sample data, which are generated through above mentioned iterative procedure. This trained network predicts strain and magnetic flux density from stress level and magnetic field. Finally, comparative study is done for linear, polynomial-nonlinear, ANN results with experimental data given in Etrema manual. The paper is then concluded with some specific observation.

2 Artificial Neural Network (ANN)

Artificial neural networks can provide non-linear parameterized mapping between a set of inputs and a set of outputs with unknown function relationship. A three-layer network (Figure-1) with the sigmoid activation functions can approximate any smooth mapping. A typical supervised feed-forward multi layer neural network is called as a back propagation (BP) neural network. The structure

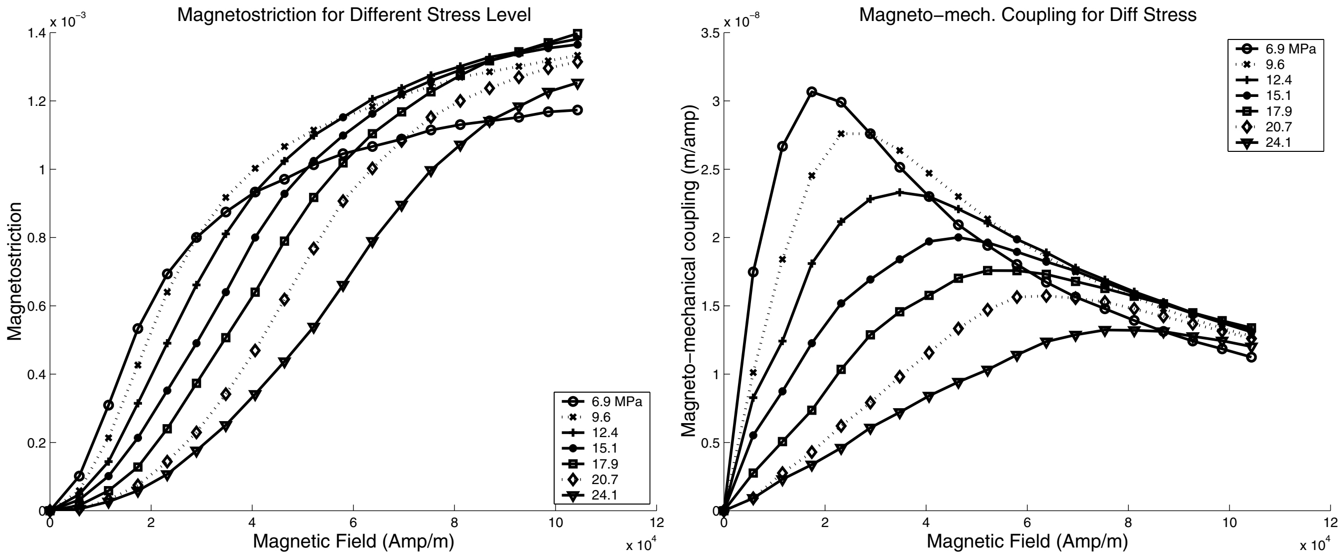

Figure 2 : Magnetostriction and magneto-mechanical coupling vs. magnetic field supplied by Etrema

of a BP neural network shown in Figure-1 mainly include an input layer for receiving the input data; some hidden layer for processing data; and an output layer to indicate the identified results. In this study, the tusk of identifying nonlinear magnetostriction through ANN is performed by training the neural network using the known samples.

2.1 Training Of Network

The training of a BP neural network is a two-step procedure [Rumelhart, Hinton, and Williams (1986)]. In the first step, the network propagates input through each layer until an output is generated. The error between the output and the target output is then computed. In the second step, the calculated error is transmitted backwards from the output layer and the weights are adjusted to minimize the error. The training process is terminated when the error is sufficiently small for all training samples. In practical applications of the back-propagation algorithm, learning is the result from many presentations of this training examples to the multi-layer perceptron. One complete presentation of the entire training set during the learning process is called an *epoch*. The learning process is maintained on an epoch-by-epoch basic until the synaptic weights and bias levels of the network stabilizes and the averaged squared error over the entire training set converges to some minimum value. For a given training set, back-propagation learning can be done in sequential or batch mode.

2.2 Validation Of Trained Network

To validate the trained network, the data set is separated into two parts, one for training and the other for testing the network performance. The network will be trained using training sample and the trained network will be validated with the test sample. A network is said to generalize well when the input-output mapping computed by the network, is corrected with the test data that was never used in creating or training the network. Although the network performs useful interpolation, because of multi-layered perceptrons with continuous activation functions, it leads to output functions that are also continuous.

3 Anhysteretic Coupled Constitutive Model.

Here experimental data is taken from Etrema manual [Butler (1988)] for Terfenol-D, a giant magnetostrictive material to verify the proposed model. Experimental data of magnetostriction vs. magnetic field for different stress level given in the manual is reproduced in Figure-2 and stress vs. strain curves for different magnetic field level is reproduced in Figure-3.

Application of magnetic field causes strain in the magnetostrictive material (Terfenol-D) and hence the stress, which changes magnetization of the material. As described by Butler (1988), Moffett, Clark, Wun-Fogle, Linberg, Teter, and McLaughlin (1989), and Hall and Flatau (1994), the three-dimensional coupled constitutive

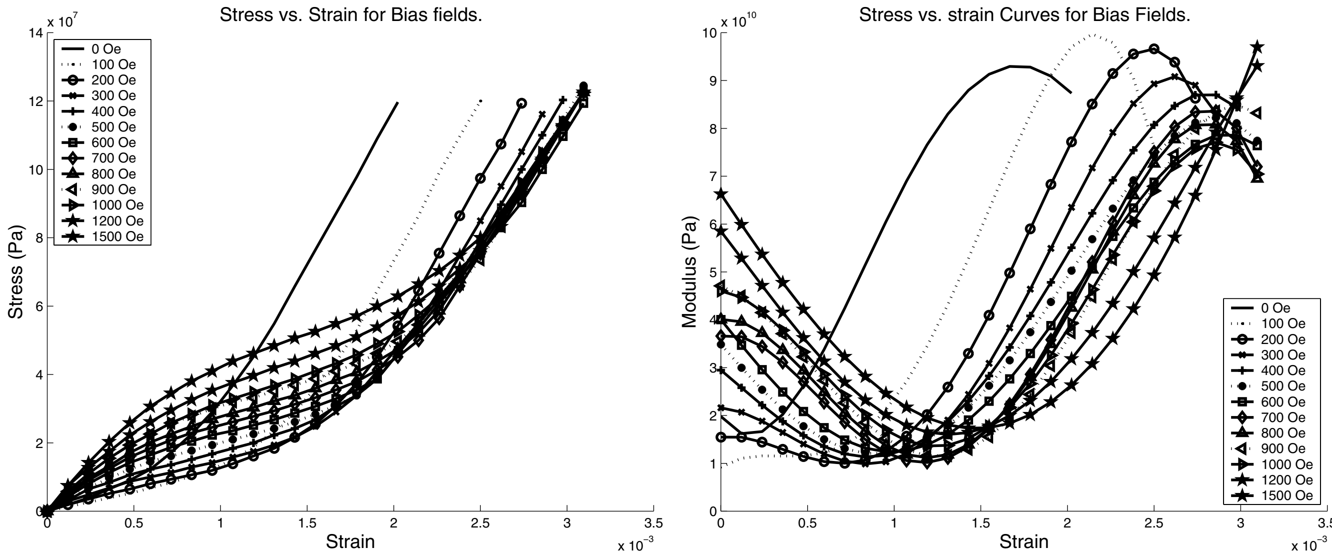

Figure 3 : stress and Q vs. strain relationship for different H level supplied by Etrema

relationship between magnetic and mechanical quantities for magnetostrictive material are given by

$$\{\varepsilon\} = [S^{(H)}]\{\sigma\} + [d]^T\{H\} \tag{1}$$

$$\{B\} = [\mu^{(\sigma)}]\{H\} + [d]\{\sigma\} \tag{2}$$

where $\{\varepsilon\}$ and $\{\sigma\}$ are strain and stress respectively. $[\mathbf{S}^{(\mathbf{H})}]$ represents elastic compliance measured at constant $\{\mathbf{H}\}$ and $[\mu^{(\sigma)}]$ represents the permeability measured at constant stress $\{\sigma\}$. Here **[d]** is the magneto-mechanical coupling coefficient, which provides a measure of the coupling between the mechanical strain and magnetic field. In general, $[\mathbf{S}]$, $[\mathbf{d}]$ and $[\mu]$ are nonlinear as they depend upon $\{\sigma\}$ and $\{\mathbf{H}\}$.

Equation-(1) is often referred to as the direct effect and Equation-(2) is known as the converse effect. These equations are traditionally used for actuation and sensing purpose, respectively. It should be noted that the elastic constants used, correspond to the fixed magnetic field values and the permeability correspond the fixed stress values.

3.1 Coupled Constitutive Model.

Analysis of smart structures using magnetostrictive materials as either sensors or actuators has traditionally been performed using uncoupled models. Uncoupled models make the assumption that the magnetic field within the magnetostrictive material is constant and proportional to the electric coil current times the number of coil turn per unit length [Ghosh and Gopalakrishnan (2004)]. Hence actuation and sensing problems are solved by two uncoupled equations, which are given by the last part of equations (1) and (2), respectively. This makes the analysis relatively simple, however this method has its limitations. It is quite well known that $[\mathbf{S}]$, $[\mathbf{d}]$ and $[\mu]$ all depend on stress level and magnetic field. In the presence of mechanical loads, the stress changes and so is the magnetic field. Estimating the constitutive properties using uncoupled model in such cases will give inaccurate predictions. Hence, the constitutive model should be represented by a pair of coupled equations given by Equation (1) & (2) to predict the mechanical and magnetic response. It is therefore necessary to simultaneously solve for both the magnetic response as well as the mechanical response regardless of whether the magnetostrictive material is being used as a sensor or actuator. Due to in-built non-linearity, the uncoupled model may not be capable of handling certain applications such as (1) modelling passive damping circuits in vibration control and (2) development of self-sensing actuators in structural health monitoring. In these applications, the coupled equations requires to be solved simultaneously. The solution of coupled equations simultaneously is a necessity for general-purpose analysis of adaptive structures built with magnetostrictive materials.

In general, the errors that result from using uncoupled models, as opposed to coupled ones, are problem dependent. There are some cases where very large differences exist in situation, where an uncoupled model is used over a coupled model [Ghosh and Gopalakrishnan (2002)].

In this work, the coupled case is analyzed with both linear and non-linear model. In linear-coupled model, magneto-mechanical coefficient, elasticity matrix and permeability matrix are assumed as constant. In nonlinear-coupled model, mechanical and magnetic nonlinearity are decoupled in their respective domains. The nonlinear stress-strain relationship is generally represented by modulus of elasticity and the nonlinear magnetic flux-magnetic field relationship represented by permeability of the material. Magneto-mechanical coupling coefficient will be assumed as constant in this case.

3.1.1 Linear Model:

From Equation-(1) and Equation-(2), the 3D constitutive model for the magnetostrictive material can be written as

$$\{\sigma\} = [Q]\{\varepsilon\} - [e]^T\{H\} \tag{3}$$

$$\{B\} = [e]\{\varepsilon\} + [\mu^{\varepsilon}]\{H\} \tag{4}$$

Where $[\mathbf{Q}]$ is Elasticity matrix, which is the inverse of compliance matrix $[\mathbf{S}]$, $[\mu^{\varepsilon}]$ is the permeability at constant strain. $[\mu^{\varepsilon}]$ and $[\mathbf{e}]$ are related to $[\mathbf{Q}]$ through

$$[e] = [d][Q] \tag{5}$$

$$[\mu^{\varepsilon}] = [\mu^{\sigma}] - [d][Q][d]^T \tag{6}$$

For ordinary magnetic materials, where magnetostrictive coupling coefficients are zero, $[\mu^{\varepsilon}]$=$[\mu^{\sigma}]$, the permeability.

Consider a magnetostrictive rod element of length L, area A, with Young modulus Q. If a tensile force F is applied the rod develops a strain ε, and hence a stress σ. Total strain energy in the rod will be

$$\begin{aligned} V_e &= \frac{1}{2}\int \varepsilon\sigma dv = \frac{1}{2}\int \varepsilon\{Q\varepsilon - eH\}dv \\ &= \frac{1}{2}\int \varepsilon Q\varepsilon dv - \frac{1}{2}\int \varepsilon eH dv \\ &= \frac{1}{2}ALQ\varepsilon^2 - \frac{1}{2}ALe\varepsilon H \end{aligned} \tag{7}$$

Magnetic potential energy in magnetostrictive rod is

$$\begin{aligned} V_m &= \frac{1}{2}\int BH dv = \frac{1}{2}\int \{e\varepsilon + \mu^{\varepsilon}H\}H dv \\ &= \frac{1}{2}\int \varepsilon eH dv + \frac{1}{2}\int H\mu^{\varepsilon}H dv \\ &= \frac{1}{2}ALHe\varepsilon + \frac{1}{2}AL\mu^{\varepsilon}H^2 \end{aligned} \tag{8}$$

Magnetic external work done for N number of coil turn with coil current I is

$$W_m = IN\mu^{\sigma}HA \tag{9}$$

Mechanical External work done is

$$W_e = F\varepsilon L \tag{10}$$

Total potential energy of the system comes $T_p = -(V_e - W_e) + (V_m - W_m)$.

$$\begin{aligned} T_p = &-\frac{1}{2}ALQ\varepsilon^2 + \frac{1}{2}ALe\varepsilon H \\ &+\frac{1}{2}ALHe\varepsilon + \frac{1}{2}AL\mu^{\varepsilon}H^2 - IN\mu^{\sigma}HA + F\varepsilon L \end{aligned} \tag{11}$$

Using Hamilton's Principle, $\delta(\int_{t_1}^{t^2} T_p\, dt) = 0$ two equations in terms of H and ε, will come.

$$-ALQ\varepsilon + ALeH + FL = 0 \tag{12}$$

$$ALe\varepsilon + ALH\mu^{\varepsilon} - IN\mu^{\sigma}A = 0 \tag{13}$$

Dividing both equations by AL, equations will be

$$-Q\varepsilon + eH = -\frac{F}{A} \tag{14}$$

$$e\varepsilon + H\mu^{\varepsilon} = \frac{IN\mu^{\sigma}}{L} \tag{15}$$

As right hand side of Equation-15 is not function of ε and left hand side is magnetic flux density (Equation-4), the magnetic flux density in this model is not function of ε. Hence, it is preserving the flux line continuity.

Eliminating H from Equation-14 and substituting this in Equation-15, stress - strain relationship of the magnetostrictive material can be written as follows.

$$H = (Q\varepsilon - F/A)/e \tag{16}$$

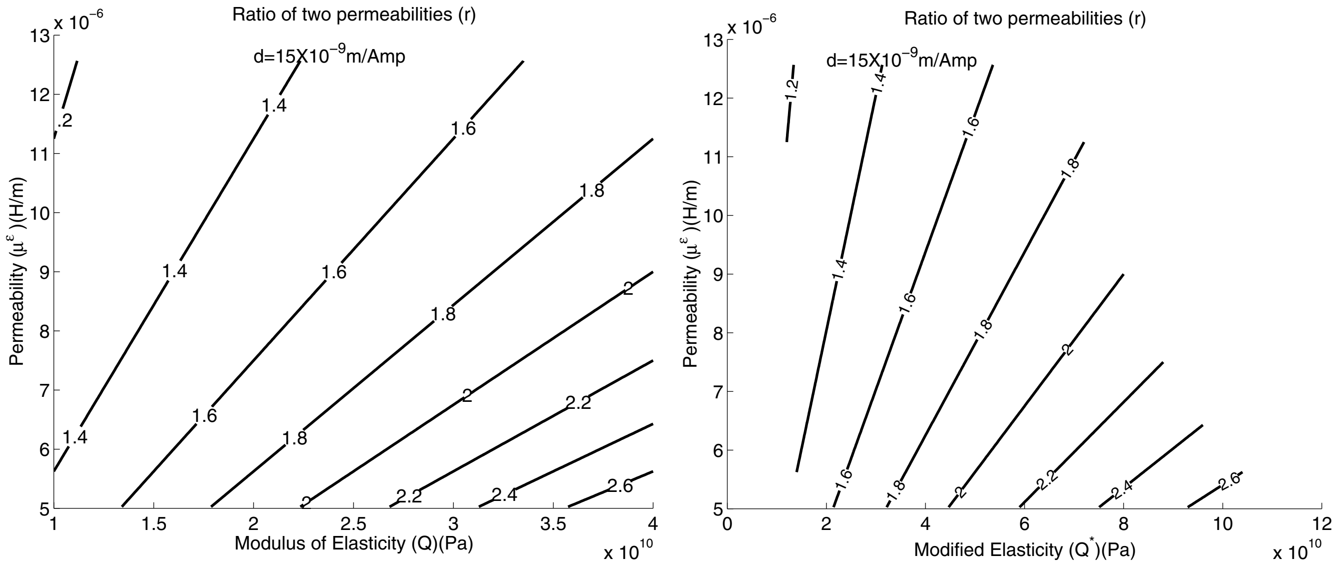

Figure 4 : Ratio of two permeabilities (r) with different values of permeability vs. modulus of elasticity (left) and modified elasticity (right), considering d=$15X10^{-9}$ m/Amp .

$$\varepsilon = \frac{IN\mu^{\sigma}Ae + L\mu^{\varepsilon}F}{ALe^2 + AL\mu^{\varepsilon}Q} = \frac{IN\mu^{\sigma}eA + F\mu^{\varepsilon}L}{AL\mu^{\sigma}Q} \tag{17}$$

From Equation-17, total strain for applied coil current I and tensile stress F/A can be written as

$$\varepsilon = \lambda + \varepsilon_{\sigma} \tag{18}$$

where λ is the strain due to coil current, which is called the magnetostriction, and ε_{σ} is the strain due to tensile stress (elastic strain).

$$\lambda = \frac{IN\mu^{\sigma}Ae}{AL\mu^{\sigma}Q} = INd/L \tag{19}$$

$$\varepsilon_{\sigma} = \frac{L\mu^{\varepsilon}F}{AL\mu^{\sigma}Q} = \frac{F}{AQ^*} \tag{20}$$

Let Q^* be the modified elastic modulus and substituting the value of e and μ^{ε} from Equation-5 and Equation-6, Q^* will be

$$Q^* = \frac{Q\mu^{\sigma}}{\mu^{\varepsilon}} = \frac{Q\mu^{\sigma}}{\mu^{\sigma} - d^2Q} = Q + \frac{e^2}{\mu^{\varepsilon}} \tag{21}$$

If the value of μ^{σ} is much greater than d^2Q, μ^{ε} can be assumed equal to μ^{σ} and Q^* can be assumed as equal to Q. If the value of μ^{σ} is much greater than d^2Q the total strain of the rod will be same as for the uncoupled model. The first term in the above expression is the strain due to magnetic field, and the second term is the strain due to the applied mechanical loading. However, for Terfenol-D [Butler (1988)], the value of d^2Q is comparable with μ^{σ}. Substituting the value of strain from Equation-17 in the Equation-16, the value of magnetic field will be

$$H = \frac{F}{Ae}\left(1 - \frac{\mu^{\varepsilon}}{\mu^{\sigma}}\right) + \frac{IN}{L} \tag{22}$$

Note that although the magnetostriction value (INd/L) in Equation-19 is same for coupled and uncoupled case, the value of magnetic field is different.

Assuming r as the ratio of two permeabilities or two elastic module. From Equation-(21), r can be written as.

$$r = \frac{\mu^{\sigma}}{\mu^{\varepsilon}} = \frac{Q^*}{Q} \tag{23}$$

If the value of r is one, result of coupled analysis is similar with uncoupled analysis. In Figure-4, the value of r is shown in contour plot for different values of constant strain permeability and modulus of elasticity considering coupling coefficient as $15X10^{-9}$ m/Amp. In the left figure, value of r is shown for different values of permeability and elastic modulus. In the right figure, value of r is shown for different values of permeability and modified elasticity. In these plots it is clear that for a particular value of elasticity if the value of permeability increase,

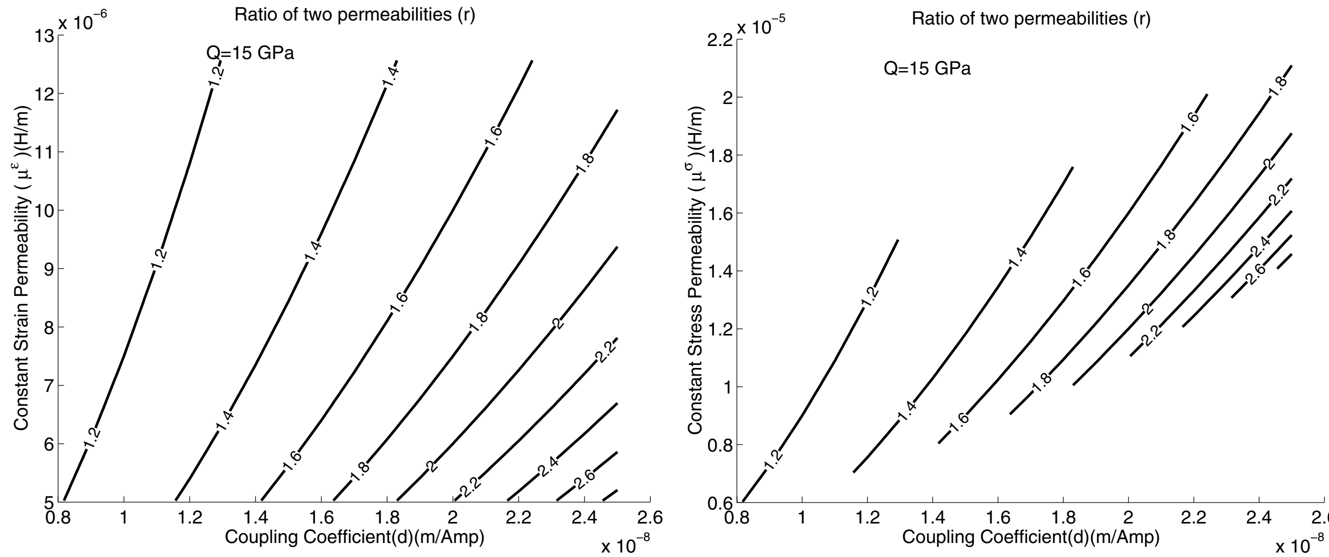

Figure 5 : Ratio of two permeabilities (r) with different values of coupling coefficient vs. constant strain permeability (left) and constant stress permeability (right), considering Q=15GPa.

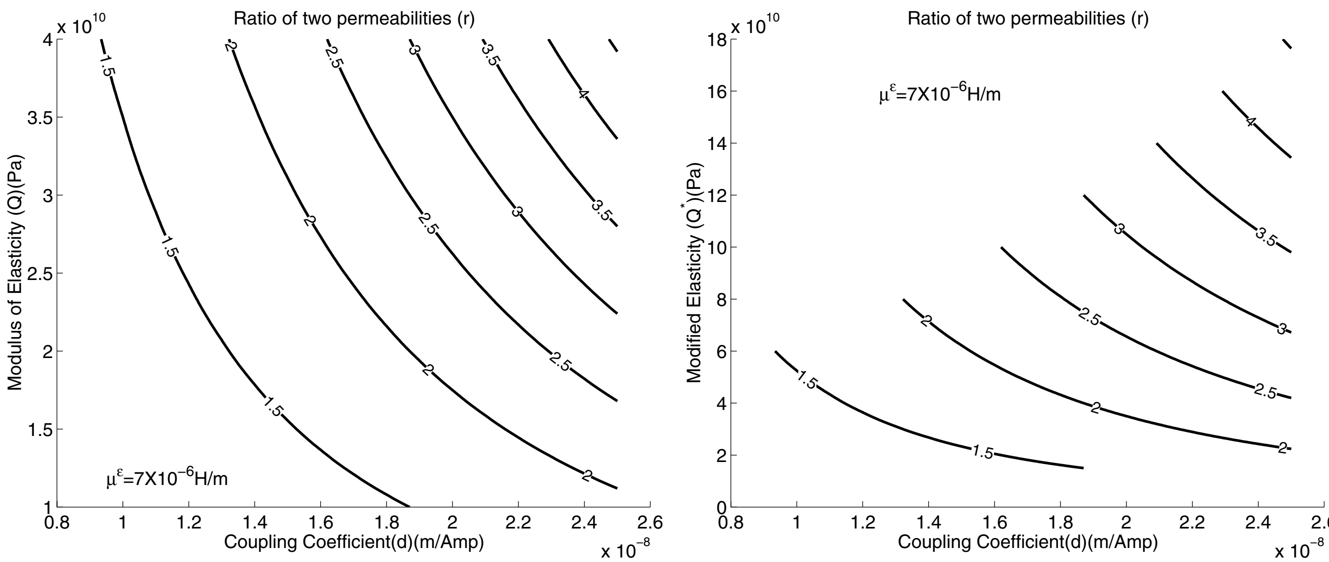

Figure 6 : Ratio of two permeabilities (r) with different values of coupling coefficient vs. modulus of elasticity (left) and modified elasticity (right), considering $\mu^{\varepsilon} = 7X10^{-6}$ henry/m.

the value of r will decrease. But for a particular value of permeability, if the value of elasticity increase the value of r will increase. In Figure-5, the value of r is shown in contour plot for different value of permeabilities and coupling coefficient considering module of elasticity as 15GPa. In the left figure, value of r is given for different values of constant strain permeability and coupling coefficient. In the right figure, value of r is shown for different values of constant stress permeability and coupling coefficients. In these plots it is clear that for a particular value of permeability, if the value of coupling coefficient increase the value of r will increase. But for a particular value of coupling coefficient if the value of permeability increase, the value of r will decrease. In Figure-6, the value of r is shown in contour plot for different value of elasticities and coupling coefficient considering constant strain permeability as $7X10^{-6}$ henry/m. In the left figure, value of r is given for different values of modulus of elasticities and coupling coefficient. In the right figure, value of r is shown for different values of modified elasticities and coupling coefficients. In these plots it is clear that for a particular value of elasticity, if the value of coupling coefficient increase the value of r will increase. Similarly, for a particular value of coupling coefficient if the value of elasticity increase, the value of r will increase.

From experimental data given in Etrema manual [Butler (1988)], the best value of Q, μ^{σ} and d is calculated, which will minimize the difference between experimental data and the data according to Equation-(17) by least square approach. In the first set of experimental data, only magnetostriction values was reported, which is expressed in Equation-(19). The value of coupling coefficient is calculated minimizing the total square error, λ_{Error}.

$$\lambda_{Error} = \sum (\lambda_{exp} - \lambda)^2 \tag{24}$$

Similarly in the second set of experimental data, strain due to compressive stress (ε_{σ}) was reported. The expression for the value of elastic strain, ε_{σ} is given in Equation-(20). In this expression, the value of Q^* is calculated minimizing the total square error $\varepsilon_{\sigma}^{Error}$.

$$\varepsilon_{\sigma}^{Error} = \sum (\varepsilon_{\sigma}^{exp} - \varepsilon_{\sigma})^2 \tag{25}$$

From Equation-(24), using first set of experimental data (plotted in Figure-2), the value of d was calculated as $14.8X10^{-9}$ (m/amp). From Equation-25, using the second set of experimental data (plotted in Figure-3), the value of Q^* is 33.4 GPa. Assuming constant strain permeability (μ^{ε}) of the material is $7X10^{-6}$ henry/m, the value of r is 1.6, constant stress permeability (μ^{σ}) is $11.2X10^{-6}$ henry/m and Q is 20.8 GPa. From these study it is clear that for giant magnetostrictive material, like Terfenol-D, coupled analysis will give better result than uncouple analysis. But for magnetostrictive material with low coupling coefficient, the uncouple analysis will give similar result with couple analysis.

The coupled linear model cannot model the high nonlinearity of magnetostriction λ, which is required for design of actuator. Even considering nonlinear magnetic (magnetic field-magnetic flux) and mechanical (stress-strain) relationships with linear coupling coefficient, nonlinear relationships of magnetostriction cannot be modelled as it is a function of coil current, coil turn per unit length of actuator and magneto-mechanical coefficient (Equation-19). In the next section, we introduce a nonlinear model with a constant coupling coefficient, which can model the non-linear constitutive model exactly for constant magnetic coupling.

3.1.2 Nonlinear coupled model.

The model developed in this section is based on a coupled magneto-mechanical formulation, which allows accurate prediction of both the mechanical and the magnetic response of a magnetostrictive device with nonlinear magnetic and mechanical properties. Non-linearity in this model is introduced using two nonlinear curves, one for stress-strain relation and the second for magnetic field-magnetic flux relation, which enables to decouple the non-linearity in mechanical and magnetic domains. Magneto-mechanical coefficient is considered as a real parameter scalar value. Two-way coupled magneto-mechanical theory is used to model magnetostrictive material. The formulation starts with the constitutive relations. In earlier coupled-linear model, stress (σ) and magnetic flux density (B) was expressed as a function of the components of strain (ε) and magnetic field (H) as per Equation-(3) and Equation-(4). Main draw back with such an approach is that the non-linearity between magnetic domain (μ^{ε}) and mechanical domain (Q)are not uncoupled. Hence, it is difficult to model non-linearity in earlier representation. To address these issues, a different approach is used in which Equation-(3) and Equation-(4) are rearranged in terms of the mechanical strain (ε) and the magnetic flux density (B). In doing so, the mechanical non-linearity is limited to stress-strain relationship and magnetic non-linearity is limited to magnetic field-magnetic flux relationship.

One-dimensional nonlinear modelling is again studied using one-dimensional experimental data from Etrema manual. The constitutive equation can now be rewritten in terms of magnetic flux density (B) and strain (ε), as

$$\sigma = E\varepsilon - f^T B \tag{26}$$

$$H = -f\varepsilon + gB \tag{27}$$

Where

$$\begin{aligned} g &= (\mu^{\varepsilon})^{-1} \\ f &= gdQ = e/\mu^{\varepsilon} \\ E &= Q + Qdf = Q^* \end{aligned} \tag{28}$$

Like linear case, considering a magnetostrictive rod element of length L, area A, applied tensile force F, strain ε, stress σ, elastic modulus E. Total strain energy in the rod will be

$$V_e = \frac{1}{2}AL\varepsilon\sigma = \frac{1}{2}\varepsilon(E\varepsilon - fB) = \frac{1}{2}ALE\varepsilon^2 - \frac{1}{2}AL\varepsilon fB \quad (29)$$

Magnetic potential energy in magnetostrictive rod is

$$V_m = \frac{1}{2}ALBH = \frac{1}{2}AL(-f\varepsilon + gB)H = -\frac{1}{2}ALBf\varepsilon + \frac{1}{2}ALgB^2 \quad (30)$$

Magnetic external work done for N turn coil with coil current I is

$$W_m = INBA \quad (31)$$

Mechanical External work done is

$$W_e = F\varepsilon L \quad (32)$$

Total potential energy of the system is equal to $Tp = -Ve - Vm + Wm + We$.

$$T_p = -\frac{1}{2}ALE\varepsilon^2 + AL\varepsilon fB - \frac{1}{2}ALgB^2 + INBA + F\varepsilon L \quad (33)$$

Using Hamilton's Principle like linear model, two equation of B and ε will be get

$$-ALE\varepsilon + ALfB + FL = 0 \quad (34)$$

$$AL\varepsilon f - ALgB + INA = 0 \quad (35)$$

Dividing by Volume, AL Equation-(34) and Equation-(35) will become

$$E\varepsilon - fB = \frac{F}{A} \quad (36)$$

$$-f\varepsilon + gB = \frac{IN}{L} \quad (37)$$

Eliminating B from Equation-(36) and substituting this in Equation-(37), stress-strain relationship for the magnetostrictive material can be obtained.

$$B = \frac{E\varepsilon - F/A}{f} \quad (38)$$

$$\varepsilon = \frac{F/A + INf/(gL)}{E - f^2/g} \quad (39)$$

Assuming E^* as the magnetically free elastic modulus

$$E^* = E - f^2/g = Q \quad (40)$$

Total strain for applied coil current I and tensile force, F will be

$$\varepsilon = \frac{INf}{(gLE^*)} + \frac{F}{AE^*} \quad (41)$$

Here $E = Q^*$ is the elastic modulus for a magnetically stiffened rod and $Q = E^*$ is for magnetically flexible rod. Magnetically stiffened means that the magnetic flux $B=0$ in side the rod as rod is wound by short circuited coils. Magnetically flexible means the rod is free from any coil. E-Q relation can be obtained form Equation-(28).

To model the one-dimensional nonlinear magnetostrictive stress-strain and magnetic field-magnetic flux relationships, Equation-(26) and Equation-(27) can be written as,

$$E(\varepsilon) - fB = \sigma \quad (42)$$

$$-f\varepsilon + g(B) = \frac{IN}{L} \quad (43)$$

Where f is the real parameter of scalar value, and ε - $E(\varepsilon)$, B - $g(B)$ are two real parameter nonlinear curves. The basic advantage of this model is that only two nonlinear curves are required for representing nonlinearity reported in different stress levels. As opposed to this approach, in straight forward polynomial representation of magnetostriction [KrishnaMurty, Anjanappa, Wang, and Chen (1999)] one requires single nonlinear curve for every stress level. To get the coefficients of two nonlinear curves and the value of real parameter f, experimental data from Etrema manual [Butler (1988)] is used. From strain, applied coil current and stress level available in the manual, these coefficients are evaluated. Considering modulus elasticity as 30GPa, and f as $75.3X10^6$ m/Amp as an initial guess, the values of magnetic flux density, B are calculated from Equation-(42). Similarly from Equation-(43), values of $g(B)$ are evaluated. From these B and $g(B)$ values, curves of $B - g(B)$ is computed. This curve is used to get mechanical relationship. Here, the value of B is computed from $B - g(B)$ relationship. And from this value of B, using Equation-(42), values of

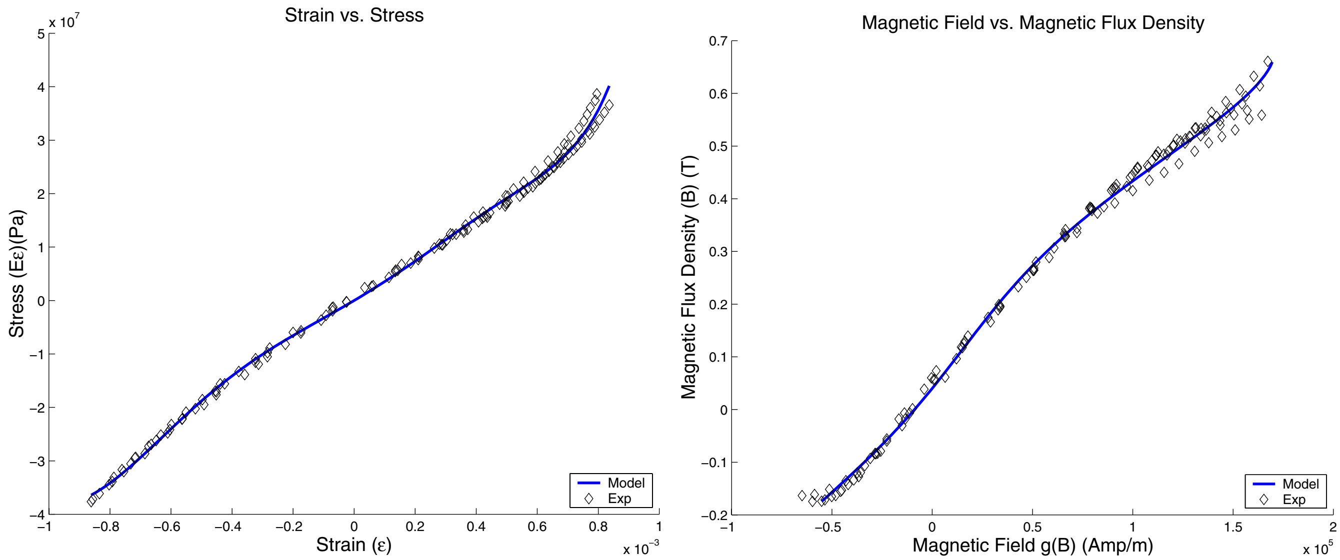

Figure 7 : Stress-Strain and magnetic field-flux curves.

Table 1 : Coefficients for sixth order polynomial.

	c	d	a	b
6	0	0	4.5419e+28	1.5853e-50
5	-2.1687e+06	-1.9526e-27	7.6602e+25	-6.1288e-44
4	1.5211e+06	1.1589e-21	-4.2662e+22	-1.0355e-34
3	3.5828e+05	-2.0047e-16	-6.6788e+19	-4.0508e-27
2	-2.1062e+05	2.0096e-12	2.3911e+16	1.0806e-19
1	2.2754e+05	4.7789e-06	1.2539e+13	2.7977e-11
0	-8.8129e+03	4.4239e-02	3.3893e+10	-1.9704e-05

$E(\varepsilon)$ is calculated. From the $E(\varepsilon)$ and ε values, the mechanical nonlinear curve of $\varepsilon - E(\varepsilon)$ relationship is computed. In summery, first the magnetic nonlinear curve is evaluated from mechanical nonlinear curve and mechanical nonlinear curve is evaluated from magnetic nonlinear curve with the help of experimental data given in Etrema manual [Butler (1988)]. This iteration will continue till both mechanical curve and magnetic curve converges. Thus, with the help of experimental data given in Etrema manual [Butler (1988)] and Equations-(42) and (43), nonlinear mechanical and magnetic relationship is evaluated. Initial values of modulus of elasticity and f are computed on trial and error basis.

For sensor device, where coil current is assumed as zero, strain and the value of magnetic flux due to the application of stress is given by

$$\varepsilon = \frac{g(B)}{f} \tag{44}$$

$$B = \frac{E(\varepsilon) - \sigma}{f} \tag{45}$$

Nonlinear curves for magnetic and mechanical properties are shown in Figure-7. These two nonlinear curves are represented as sixth order polynomial given in Equation-(46) and Equation-(47). Coefficients of these polynomials curves are given in Table-1, where unit of B is tesla, $g(B)$ is Amp/m and $E(\varepsilon)$ is Pa. The value of magneto-mechanical coupling parameter (f) is $75.3X10^6$ m/amp, which is reciprocal of $13.3X10^{-9}$ amp/m.

$$\begin{aligned} g(B) &= c_5 * B^5 + .. + c_1 * B + c_0 \\ B &= d_5 * g(B)^5 + .. + d_1 * g(B) + d_0 \end{aligned} \tag{46}$$

$$\begin{aligned} E(\varepsilon) &= a_6 * \varepsilon^6 + .. + a_1 * \varepsilon + a_0 \\ \varepsilon &= b_6 * E(\varepsilon)^6 + .. + b_1 * E(\varepsilon) + b_0 \end{aligned} \tag{47}$$

On the basis of these two curves given in Equation-(46) and Equation-(47) and parameter (f), strain, mag-

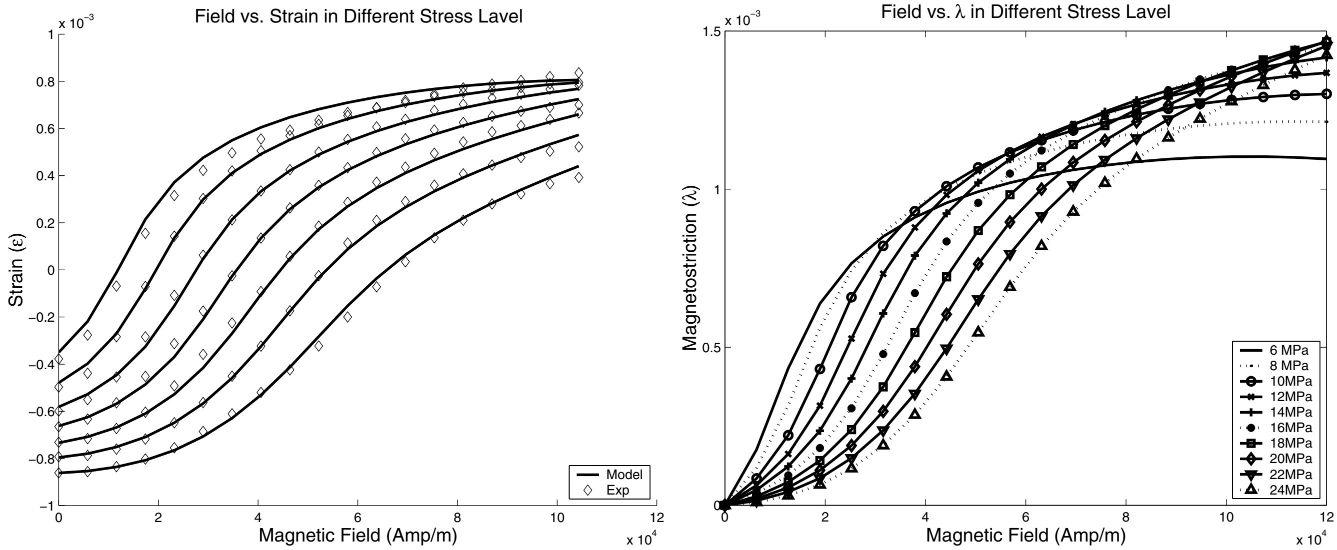

Figure 8 : Magnetic field-strain in different stress level.

netostriction vs. applied magnetic field for different stress level are plotted in Figure-8. The experimental data of strain-magnetic field relationships for different stress level is almost matching with this model. Similarly strain-compressive force and elastic modulus for different magnetic field level is plotted in Figure-9. Elastic modulus is initially decreases and then increases for each magnetic field level, which is also reported in Butler (1988).

Calculation of Flux and Strain from Coil Current and Stress: As two nonlinear curves are related in these relationships, the calculation of magnetic flux and strain from stress and coil current is an iterative procedure. Initially, the value of magnetic flux, B is assumed a certain value. From $B-g(B)$ curve, the value of $g(B)$ is evaluated. From Equation-(37) the value of strain is evaluated considering magnetic field as coil current times coil turn per unit length of actuator. Using this strain, from the $\varepsilon - E(\varepsilon)$ curve the value of $E(\varepsilon)$ can be found. From Equation-(36), the value of B can be determined. If this B value is not same as assumed, this iteration will be continued until the value converges.

3.1.3 ANN model.

To avoid iterative procedure mentioned in nonlinear model, one three layer artificial neural network is developed, which gives direct nonlinear mapping from magnetic field and stress to magnetic flux density and strain. Standard logistic function $y = 1/(1+e^{-1.7159v})$ is used in hidden layer as activation function with linear output layer. Input (stress and magnetic field) and output (strain and magnetic flux density) data is normalized for better performance of network.

$$\sigma_n = \frac{(\sigma - \sigma_{mean})}{(max|\sigma| - \sigma_{mean})} \tag{48}$$

$$H_n = \frac{(H - H_{mean})}{(max|H| - H_{mean})} \tag{49}$$

$$\varepsilon_n = \frac{(\varepsilon - \varepsilon_{mean})}{(max|\varepsilon| - \varepsilon_{mean})} \tag{50}$$

$$B_n = \frac{(B - B_{mean})}{(max|B| - B_{mean})} \tag{51}$$

σ_n, the normalized stress is calculated using Equation-(48). The value of σ_{mean} and $(max|\sigma| - \sigma_{mean})$ are $-1.57966X10^7$ and $0.830345X10^8$ Pa respectively. Similarly H_n is normalized magnetic field, is calculated from Equation-(49). The value of H_{mean} and $(max|H| - H_{mean})$ in Equation-(49) are both 750 Oe. Normalized strain ε_n is the output of network, from which strain is calculated using Equation-(50). The value of ε_{mean} and $(max|\varepsilon| - \varepsilon_{mean})$ are $0.203245X10^{-03}$ and $0.106504X10^{-02}$ respectively. In Equation-(51) the value of B_{mean} is 0.385126

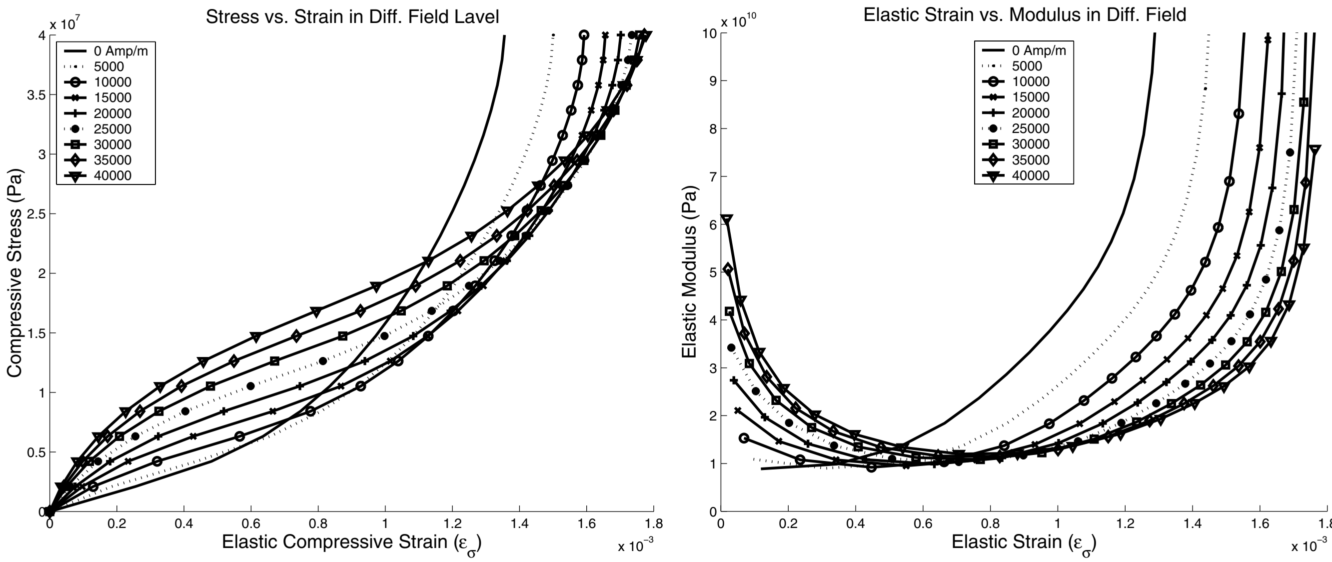

Figure 9 : Stress and modulus vs. strain curves for different field level.

tesla and the value of $(max|B| - B_{mean})$ are 0.319818 and 0.499656 tesla respectively.

Table 2 : Connection between input layer and hidden layer.

Input Layer Neurons	Hidden Node 1	Hidden Node 2	Hidden Node 3	Hidden Node 4
H_n	4.6187	-2.1241	-.15066	.38726
σ_n	1.2866	-1.3195	-.43318	.031765
Input Bias	2.2483	-.25721	-.61579	.088719

To train this network some training and validation samples are generated through iterative process stated earlier. Weight and bias parameter of the trained network is given in Table-2 and Table-3. Different validation studies are also carried out.

Table 3 : Connection between hidden layer and output layer.

Output Layer Neurons	ε_n	B_n
Hidden Node 1	.63447	.67409
Hidden Node 2	-.53313	-.23172
Hidden Node 3	-.69546	-.071654
Hidden Node 4	.48235	2.1900
Output Bias Node	-.29930	-1.5467

3.2 Comparison Between Different Coupled Models.

Comparative study of different models are done taking a magnetostrictive rod with varying magnetic field and stress level. Three different stress levels (6.9 MPa,15.1 MPa and 24.1MPa) are taken to compute the total strain and magnetic flux density in the rod for varying magnetic field level and shown in Figure-10. In the left Figure, total strain is shown according to linear, polynomial, ANN and experimental approaches. Both polynomial and ANN approach is showing close result with the experimental data throughout the magnetic field range. Where as in linear model, results are not matching with the experimental data throughout the magnetic field range. But, this model can be used in low magnetic field level for medium stress level and in medium field level for high stress level. For low stress level, linear model can be used on an average sense. In the absence of experimental data (Etrema manual) of magnetic flux density, only computational results are shown in right figure. Magnetic flux density is shown according to linear, polynomial and ANN approach. Similar to strain result, results of ANN model and polynomial model are in excellent agreement. However, the results of linear model is not matching through out the magnetic field range. In linear model, for medium stress level in low magnetic field level magnetic flux density is matching with the nonlinear model. For high and low stress level, linear model

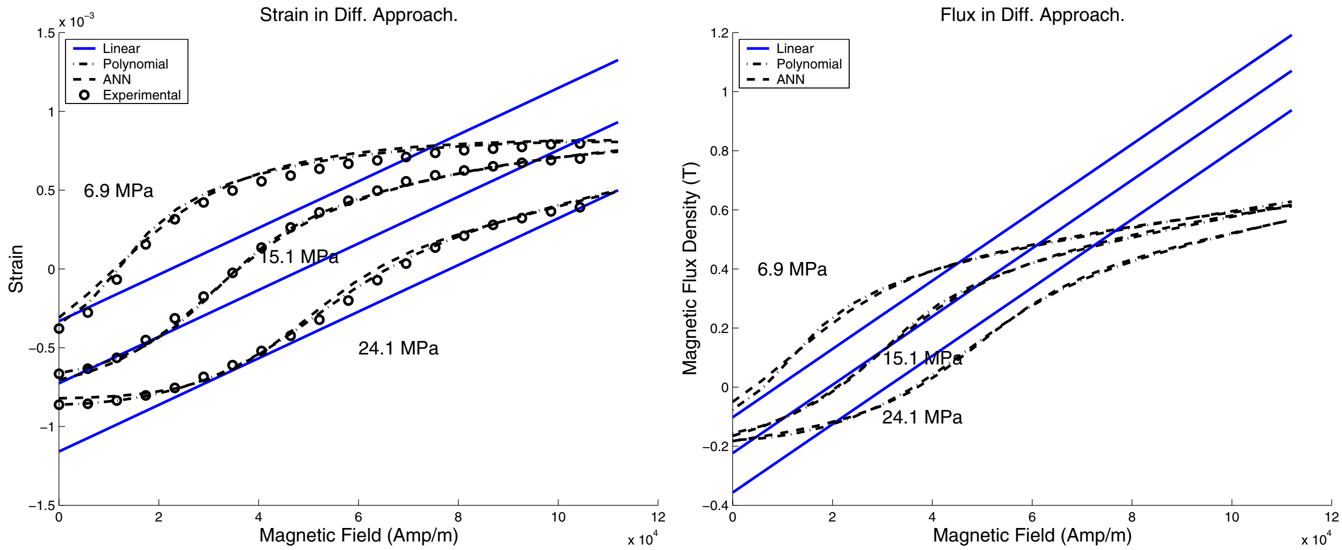

Figure 10 : Strain and magnetic flux density for different stress level.

can be used in an average sense.

4 Conclusions

This study is mainly intended for anhysteretic linear and nonlinear, coupled constitutive relationship of magnetostrictive material. Coupled model is studied without assuming any direct relationship of magnetic field unlike uncoupled model. In linear-coupled model elastic modulus, permeability and magnetoelastic constant is considered as constant. But this model cannot predict the highly nonlinear properties of magnetostrictive material. In nonlinear-coupled model, nonlinearity is decoupled in magnetic domain and mechanical domain using two nonlinear curves for stress-strain and magnetic flux density-magnetic field intensity. In this model, the computation of magnetostriction requires the value of magnetic flux density, which comes through an iterative process for nonlinearity of curves. To avoid this iterative computation one three layer artificial neural network is developed, which will give nonlinear mapping from stress level and magnetic field to strain and magnetic flux density. Finally, comparative study of linear, polynomial and ANN approach is done and shown that linear coupled model can predict the constitutive relationships in an averaged sense only. Nonlinear models are shown to predict experimental results exactly throughout the magnetic field range.

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