

# “Constraint Intensity-Driven Evolutionary Multitasking for Constrained Multi-Objective Optimization” Supplementary Materials

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## 1. Detailed results of the test suite

### 1.1 The performance of CIDEMT and the benchmark algorithm on the LIRCMOP, DASCMP and DOC problems

As shown in Tables S1–S6, the performance of the CIDEMT algorithm and six benchmark algorithms on the LIRCMOP, DASCMP, and DOC problem sets was evaluated using the IGD and HV metrics. Among the LIRCMOP problems, LIRCMOP5, LIRCMOP6, LIRCMOP9, LIRCMOP10, LIRCMOP13, and LIRCMOP14 belong to Type I problems; LIRCMOP11 and LIRCMOP12 are categorized as Type II problems; while the remaining LIRCMOP problems, as well as all DASCMP and DOC problems, fall under Type III problems.

When comparing CIDEMT against the six baseline algorithms across the three categories of multi-objective optimization problems (LIRCMOP, DASCMP, and DOC), CIDEMT demonstrates significant overall superiority based on both the IGD and HV indicators. Specifically, within the LIRCMOP problem set, CIDEMT achieves the lowest IGD values in 11 test problems, indicating excellent precision and stability in approximating the true Pareto front. In addition, CIDEMT ranks first in HV values for the majority of these problems, highlighting its ability to not only produce high-accuracy solutions but also achieve better coverage of the objective space.

However, for Type I problems such as LIRCMOP13 and LIRCMOP14, CIDEMT slightly underperforms compared to EMCMP. This may be attributed to its density estimation and selection strategies being less effective in covering all regions of the solution space when dealing with high-dimensional, multimodal Pareto fronts. Similarly, in the Type III problem LIRCMOP1, which features a narrow feasible region and multimodal characteristics, CIDEMT's performance in maintaining solution diversity was outpaced by CMOEMT.

On the DASCMP problems, CIDEMT also performs competitively, achieving optimal or near-optimal results on most test problems. Although it does not outperform MTCMP and EMCMP on DASCMP4 and DASCMP5, the performance gap is relatively small. The slight underperformance is mainly due to task T3, which, while enhancing diversity, compromises the algorithm's stability to some extent.

For the more challenging DOC problems—characterized by complex constraints and extreme objective conflicts in both the decision and objective spaces—most traditional algorithms suffer a significant drop in performance. In contrast, CIDEMT maintains robust convergence and distribution capabilities under such harsh conditions. While it does not

achieve the best result on DOC1, the shortfall may stem from the algorithm’s limited ability to handle biased distributions in the objective space.

To more intuitively illustrate CIDEMT's problem-solving capabilities, Figures S1–S3 present the solution distributions of CIDEMT and benchmark algorithms across various test problems. As seen in Figures S1 and S2, CIDEMT obtains well-distributed and converged solution sets on LIRCMOP2 and DASCOP2. Although the benchmark algorithms also yield feasible solutions, they tend to get trapped in local regions, resulting in loss of diversity. In Figure S3, for the DOC6 problem—where the objective space is heavily constrained—benchmark algorithms struggle to find feasible solutions, whereas CIDEMT exhibits outstanding performance. Although some solutions are scattered across discrete regions, CIDEMT still achieves superior overall coverage of the constrained Pareto front.

In conclusion, CIDEMT demonstrates not only excellent performance on standard multi-objective problems but also exceptional convergence, distribution balance, and stability on problems with high constraint complexity and difficulty. In the comprehensive evaluation of the LIRCMOP, DASCOP, and DOC test suites, CIDEMT consistently achieves outstanding IGD and HV values across the majority of problems, showcasing its dual strengths in solution accuracy and objective space coverage. These results validate the effectiveness of CIDEMT’s co-evolution strategy, density-guided mechanism, and constraint-handling methods.

Nevertheless, the algorithm does not always achieve optimal results on all problems, revealing areas for further enhancement. Specifically, CIDEMT’s capability in handling problems with complex boundary geometries, highly discontinuous feasible regions, or segmented/non-convex Pareto fronts requires improvement. Future work may incorporate proactive boundary exploration strategies, enhanced diversity preservation mechanisms, and adaptive parameter control techniques to bolster generalization across diverse problem structures. Through such improvements, CIDEMT is expected to achieve more comprehensive and stable performance in a broader spectrum of multi-objective optimization problems.

Table S1 Comparison results of IGD values of CIDEMT with 6 benchmark algorithms on LIRCMOP

Problem	MTCMO	CMOEMT	EMCMO	APSEA	IMCMOEAD	CMOES	CIDEMT
LIRCMOP1	9.8072e-2 (1.96e-2) -	<b>1.4250e-2 (1.02e-2) +</b>	2.0895e-1 (4.90e-2) -	2.7799e-1 (3.55e-2) -	3.1135e-1 (3.66e-2) -	1.2710e-1 (2.83e-2) -	2.5642e-2 (8.73e-3)
LIRCMOP2	9.8248e-2 (1.24e-2) -	1.9368e-2 (3.29e-2) -	1.7624e-1 (3.66e-2) -	2.3866e-1 (2.33e-2) -	2.3183e-1 (2.43e-2) -	1.1207e-1 (1.85e-2) -	<b>4.2606e-3 (1.66e-4)</b>
LIRCMOP3	1.0238e-1 (2.82e-2) -	5.7045e-3 (2.35e-3) -	2.1353e-1 (6.44e-2) -	3.0446e-1 (4.25e-2) -	3.2783e-1 (3.23e-2) -	1.0343e-1 (3.68e-2) -	<b>2.6198e-3 (1.65e-3)</b>
LIRCMOP4	1.2190e-1 (2.23e-2) -	2.9152e-2 (6.26e-2) -	2.3137e-1 (4.85e-2) -	2.8516e-1 (3.28e-2) -	3.0100e-1 (2.15e-2) -	1.3375e-1 (2.34e-2) -	<b>2.3156e-3 (1.81e-3)</b>
LIRCMOP5	2.6922e-1 (5.41e-2) -	2.6591e-2 (3.85e-2) =	2.7973e-1 (4.80e-2) -	7.0520e-1 (4.80e-1) -	1.2313e+0 (2.59e-2) -	2.7241e-1 (4.96e-2) -	<b>7.1763e-3 (6.92e-4)</b>
LIRCMOP6	3.7711e-1 (2.76e-1) -	4.0953e-2 (5.96e-2) =	2.7255e-1 (6.59e-2) -	7.7906e-1 (4.87e-1) -	1.3542e+0 (1.43e-1) -	2.9591e-1 (8.25e-2) -	<b>7.4105e-3 (4.39e-4)</b>
LIRCMOP7	9.9784e-2 (2.71e-2) -	1.5623e-2 (2.14e-2) -	1.0620e-1 (2.53e-2) -	1.0279e-1 (2.92e-2) -	1.4089e+0 (6.50e-1) -	1.0292e-1 (3.09e-2) -	<b>7.2760e-3 (1.97e-4)</b>
LIRCMOP8	1.6505e-1 (5.86e-2) -	1.5839e-2 (2.55e-2) -	1.5626e-1 (4.64e-2) -	1.4737e-1 (5.03e-2) -	1.4384e+0 (5.99e-1) -	1.4361e-1 (5.29e-2) -	<b>7.2325e-3 (1.93e-4)</b>
LIRCMOP9	5.7184e-1 (1.58e-1) -	2.4644e-1 (8.11e-2) -	3.7582e-1 (1.42e-1) -	6.3664e-1 (1.74e-1) -	7.3008e-1 (1.40e-1) -	2.8613e-1 (7.06e-2) -	<b>6.1985e-2 (2.97e-2)</b>
LIRCMOP10	1.4915e-1 (5.53e-2) -	6.3473e-2 (5.23e-2) -	8.3619e-2 (3.99e-2) -	5.6577e-1 (2.81e-1) -	7.3739e-1 (7.54e-2) -	8.0692e-2 (3.46e-2) -	<b>8.8076e-3 (5.91e-4)</b>
LIRCMOP11	1.3325e-1 (9.03e-2) -	4.5032e-2 (4.77e-2) -	5.0979e-2 (2.85e-2) -	5.3127e-1 (2.20e-1) -	4.9976e-1 (1.82e-1) -	4.3221e-2 (3.55e-2) -	<b>2.6556e-3 (1.54e-4)</b>
LIRCMOP12	2.2744e-1 (9.92e-2) -	8.4148e-2 (3.89e-2) -	1.5365e-1 (8.98e-2) -	3.0394e-1 (1.19e-1) -	5.0411e-1 (1.60e-1) -	9.9115e-2 (4.83e-2) -	<b>3.7282e-3 (3.54e-4)</b>
LIRCMOP13	1.3155e+0 (1.55e-3) -	9.8766e-2 (1.15e-3) +	<b>9.0606e-2 (8.47e-4) +</b>	1.3153e+0 (2.20e-3) -	1.3132e+0 (1.92e-1) -	9.5602e-2 (1.17e-3) +	1.0057e-1 (1.92e-3)
LIRCMOP14	1.2718e+0 (1.55e-3) -	1.0217e-1 (9.99e-4) -	<b>9.5576e-2 (9.60e-4) +</b>	1.2722e+0 (1.72e-3) -	1.1696e+0 (3.67e-1) -	9.6658e-2 (8.00e-4) =	9.7143e-2 (1.31e-3)
+/-/=	0/14/0	2/10/2	2/12/0	0/14/0	0/14/0	1/12/1	

Table S2 Comparison results of HV values of CIDEMT with 6 benchmark algorithms on LIRCMOP

Problem	MTCMO	CMOEMT	EMCMO	APSEA	IMCMOEAD	CMOES	CIDEMT
LIRCMOP1	1.8291e-1 (9.99e-3) -	<b>2.3438e-1 (3.82e-3) +</b>	1.4056e-1 (1.69e-2) -	1.2358e-1 (1.18e-2) -	1.1391e-1 (1.33e-2) -	1.7250e-1 (1.23e-2) -	2.2954e-1 (3.11e-3)
LIRCMOP2	3.0818e-1 (8.97e-3) -	3.5461e-1 (1.25e-2) -	2.7038e-1 (2.09e-2) -	2.3773e-1 (1.25e-2) -	2.4327e-1 (1.40e-2) -	3.0512e-1 (1.24e-2) -	<b>3.6073e-1 (1.49e-4)</b>
LIRCMOP3	1.6407e-1 (1.13e-2) -	2.0532e-1 (7.29e-4) -	1.2952e-1 (2.07e-2) -	1.0318e-1 (1.23e-2) -	9.3634e-2 (8.90e-3) -	1.6544e-1 (1.61e-2) -	<b>2.0818e-1 (1.21e-3)</b>
LIRCMOP4	2.6493e-1 (9.70e-3) -	3.0443e-1 (2.67e-2) -	2.1887e-1 (2.34e-2) -	1.9736e-1 (1.51e-2) -	1.8865e-1 (1.27e-2) -	2.6123e-1 (1.30e-2) -	<b>3.1766e-1 (8.24e-4)</b>
LIRCMOP5	1.6500e-1 (2.25e-2) -	2.8132e-1 (2.06e-2) -	1.6111e-1 (1.78e-2) -	9.0181e-2 (8.84e-2) -	0.0000e+0 (0.00e+0) -	1.6498e-1 (2.07e-2) -	<b>2.9107e-1 (3.20e-4)</b>
LIRCMOP6	1.1008e-1 (3.35e-2) -	1.8460e-1 (1.88e-2) =	1.2046e-1 (1.44e-2) -	6.4578e-2 (5.62e-2) -	1.9729e-3 (1.08e-2) -	1.2010e-1 (1.49e-2) -	<b>1.9600e-1 (2.25e-4)</b>
LIRCMOP7	2.5484e-1 (8.77e-3) -	2.9048e-1 (9.77e-3) -	2.5221e-1 (7.79e-3) -	2.5382e-1 (8.64e-3) -	4.8584e-2 (9.89e-2) -	2.5345e-1 (9.33e-3) -	<b>2.9447e-1 (9.05e-5)</b>
LIRCMOP8	2.4117e-1 (1.23e-2) -	2.9067e-1 (1.07e-2) -	2.4145e-1 (1.03e-2) -	2.4418e-1 (1.26e-2) -	4.2148e-2 (8.62e-2) -	2.4468e-1 (1.36e-2) -	<b>2.9452e-1 (8.88e-5)</b>

LIRCPOP9	3.1907e-1 (9.83e-2) -	4.8917e-1 (2.88e-2) -	4.2262e-1 (6.77e-2) -	2.9049e-1 (1.03e-1) -	2.3539e-1 (8.15e-2) -	4.8653e-1 (2.28e-2) -	<b>5.4768e-1 (7.49e-3)</b>
LIRCPOP10	6.3225e-1 (3.16e-2) -	6.7858e-1 (2.48e-2) -	6.6921e-1 (1.54e-2) -	3.3158e-1 (2.21e-1) -	1.5639e-1 (4.94e-2) -	6.6926e-1 (1.31e-2) -	<b>7.0461e-1 (3.69e-4)</b>
LIRCPOP11	6.3040e-1 (6.25e-2) -	6.6970e-1 (2.85e-2) -	6.7119e-1 (1.12e-2) -	3.7163e-1 (1.52e-1) -	3.4107e-1 (1.44e-1) -	6.7377e-1 (1.81e-2) -	<b>6.9388e-1 (6.65e-5)</b>
LIRCPOP12	5.1640e-1 (4.31e-2) -	5.8406e-1 (2.07e-2) -	5.4780e-1 (4.92e-2) -	4.8324e-1 (4.71e-2) -	3.4392e-1 (1.12e-1) -	5.7310e-1 (2.71e-2) -	<b>6.2012e-1 (1.48e-4)</b>
LIRCPOP13	1.0626e-4 (9.61e-5) -	5.5007e-1 (1.92e-3) +	<b>5.6011e-1 (8.80e-4) +</b>	1.3447e-4 (1.62e-4) -	9.4911e-3 (5.20e-2) -	5.5015e-1 (1.97e-3) +	5.3837e-1 (2.65e-3)
LIRCPOP14	5.0532e-4 (2.83e-4) -	5.4834e-1 (1.76e-3) -	<b>5.5467e-1 (1.28e-3) +</b>	4.9966e-4 (3.07e-4) -	4.8852e-2 (1.27e-1) -	5.5229e-1 (1.43e-3) +	5.5062e-1 (1.63e-3)
+/-/=	0/14/0	2/11/1	2/12/0	0/14/0	0/14/0	2/12/0	

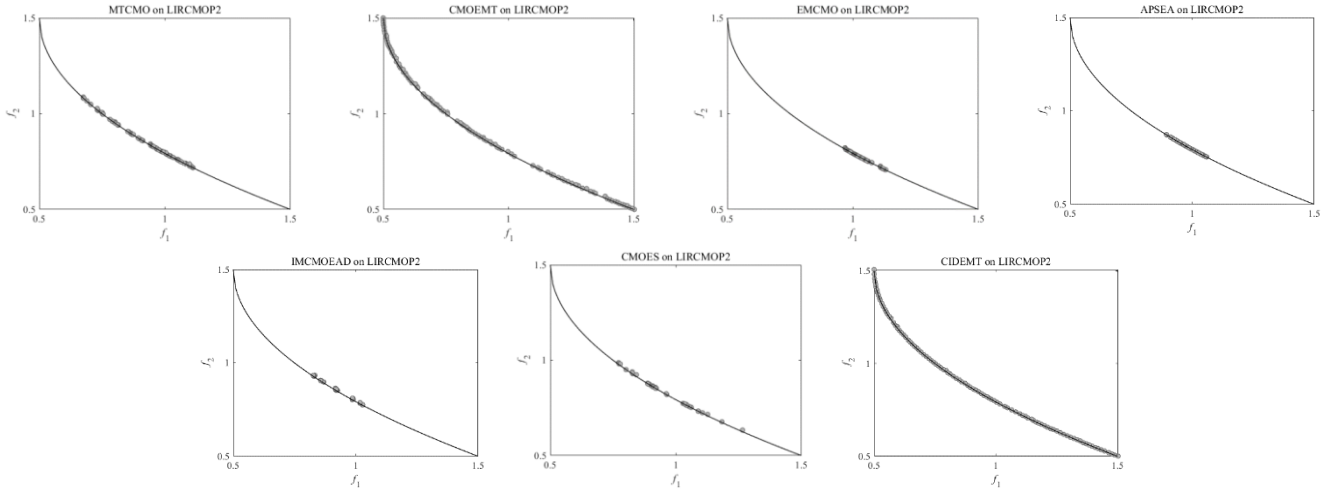


Figure S1 Distribution of final solutions of CIDEMT with 6 benchmark algorithms on LIRCPOP2

Table S3 Comparison results of IGD values of CIDEMT with 6 benchmarks on DASCPOP

Problem	MTCMO	CMOEMT	EMCMO	APSEA	IMCMOEAD	CMOES	CIDEMT
DASCPOP1	6.2723e-1 (1.05e-1) -	1.9891e-1 (2.49e-1) -	7.1026e-1 (2.80e-2) -	6.9770e-1 (4.97e-2) -	7.5578e-1 (4.40e-2) -	6.5313e-1 (7.55e-2) -	<b>2.8161e-3 (1.71e-4)</b>
DASCPOP2	2.0599e-1 (2.34e-2) -	4.4099e-3 (1.19e-4) -	2.3819e-1 (2.19e-2) -	2.4061e-1 (1.79e-2) -	4.6622e-1 (2.30e-1) -	2.2115e-1 (2.37e-2) -	<b>4.2305e-3 (8.26e-5)</b>
DASCPOP3	2.5337e-1 (3.73e-2) -	1.6054e-1 (1.45e-1) -	3.3438e-1 (3.68e-2) -	3.2776e-1 (3.35e-2) -	4.6951e-1 (2.04e-1) -	2.6237e-1 (3.34e-2) -	<b>1.4085e-2 (2.36e-3)</b>
DASCPOP4	<b>1.1586e-3 (1.24e-5) +</b>	1.3951e-3 (7.16e-4) =	1.2754e-3 (5.19e-4) -	9.1662e-3 (4.39e-2) -	5.6490e-1 (2.91e-1) -	1.1844e-3 (3.82e-5) +	1.2659e-3 (9.80e-5)
DASCPOP5	2.6995e-3 (4.05e-5) +	2.8596e-3 (1.18e-4) -	<b>2.6969e-3 (6.35e-5) +</b>	2.7051e-3 (3.21e-5) +	6.9921e-1 (2.24e-1) -	2.7423e-3 (4.98e-5) +	2.8016e-3 (7.10e-5)
DASCPOP6	1.6242e-2 (7.18e-3) -	4.0416e-2 (4.74e-2) -	1.9932e-2 (5.47e-3) -	1.5682e-1 (1.76e-1) -	5.9274e-1 (1.56e-1) -	1.7187e-2 (4.58e-3) -	<b>1.4151e-2 (2.95e-3)</b>
DASCPOP7	3.0960e-2 (8.57e-4) +	3.2431e-2 (7.83e-4) -	3.1209e-2 (6.00e-4) +	<b>3.0775e-2 (5.21e-4) +</b>	7.4295e-1 (3.46e-1) -	3.1430e-2 (6.55e-4) +	3.1791e-2 (6.67e-4)
DASCPOP8	4.0152e-2 (8.41e-4) =	4.1286e-2 (9.61e-4) -	4.0667e-2 (1.04e-3) -	4.0028e-2 (9.39e-4) =	7.1549e-1 (2.92e-1) -	3.9962e-2 (1.05e-3) =	<b>3.9838e-2 (8.74e-4)</b>
DASCPOP9	2.6067e-1 (7.36e-2) -	1.1663e-1 (1.01e-1) -	3.4257e-1 (5.83e-2) -	3.1578e-1 (6.86e-2) -	4.9684e-1 (1.58e-1) -	2.6558e-1 (9.12e-2) -	<b>3.9325e-2 (8.57e-4)</b>
+/-/=	3/5/1	0/8/1	2/7/0	2/6/1	0/9/0	3/5/1	

Table S4 Comparison results of HV values between CIDEMT and 6 benchmark algorithms on DASCPOP

Problem	MTCMO	CMOEMT	EMCMO	APSEA	IMCMOEAD	CMOES	CIDEMT
DASCPOP1	4.3940e-2 (3.33e-2) -	1.6820e-1 (5.13e-2) -	1.7055e-2 (1.96e-2) -	1.3132e-2 (8.46e-3) -	5.0166e-3 (6.17e-3) -	4.1716e-2 (2.82e-2) -	<b>2.1277e-1 (1.86e-4)</b>
DASCPOP2	2.7667e-1 (2.93e-3) -	3.5519e-1 (9.18e-5) -	2.6110e-1 (2.46e-3) -	2.5628e-1 (3.33e-3) -	1.6040e-1 (1.02e-1) -	2.6843e-1 (3.24e-3) -	<b>3.5529e-1 (6.42e-5)</b>
DASCPOP3	2.2437e-1 (1.57e-2) -	2.7049e-1 (4.52e-2) -	2.1597e-1 (1.68e-2) -	2.1510e-1 (1.53e-2) -	1.5140e-1 (9.04e-2) -	2.2347e-1 (1.45e-2) -	<b>3.1232e-1 (1.31e-4)</b>
DASCPOP4	<b>2.0432e-1 (3.57e-5) +</b>	2.0369e-1 (2.13e-3) -	2.0392e-1 (1.54e-3) -	2.0303e-1 (7.26e-3) -	3.5045e-2 (3.03e-2) -	2.0418e-1 (1.54e-4) +	2.0410e-1 (1.78e-4)
DASCPOP5	3.5177e-1 (3.85e-5) +	3.5158e-1 (9.40e-5) +	3.5167e-1 (7.09e-5) +	<b>3.5180e-1 (2.76e-5) +</b>	4.4999e-2 (3.96e-2) -	3.5169e-1 (5.33e-5) +	3.5142e-1 (1.11e-4)
DASCPOP6	3.1041e-1 (6.30e-3) =	3.0313e-1 (1.76e-2) -	3.1043e-1 (5.29e-3) -	2.3526e-1 (1.02e-1) -	5.0905e-2 (4.45e-2) -	3.1147e-1 (3.53e-3) -	<b>3.1247e-1 (7.43e-5)</b>
DASCPOP7	2.8873e-1 (3.33e-4) +	2.8758e-1 (6.72e-4) =	2.8866e-1 (4.62e-4) +	<b>2.8893e-1 (2.99e-4) +</b>	4.9893e-2 (4.99e-2) -	2.8811e-1 (4.83e-4) +	2.8748e-1 (3.98e-4)
DASCPOP8	2.0750e-1 (4.88e-4) +	2.0614e-1 (6.48e-4) =	2.0741e-1 (4.38e-4) +	<b>2.0755e-1 (3.66e-4) +</b>	2.3797e-2 (3.59e-2) -	2.0685e-1 (4.17e-4) +	2.0596e-1 (4.18e-4)
DASCPOP9	1.5031e-1 (1.40e-2) -	1.8470e-1 (2.58e-2) -	1.3381e-1 (9.83e-3) -	1.3975e-1 (1.25e-2) -	1.0733e-1 (2.96e-2) -	1.4934e-1 (1.83e-2) -	<b>2.0621e-1 (3.42e-4)</b>
+/-/=	4/4/1	1/6/2	3/6/0	3/6/0	0/9/0	4/5/0	

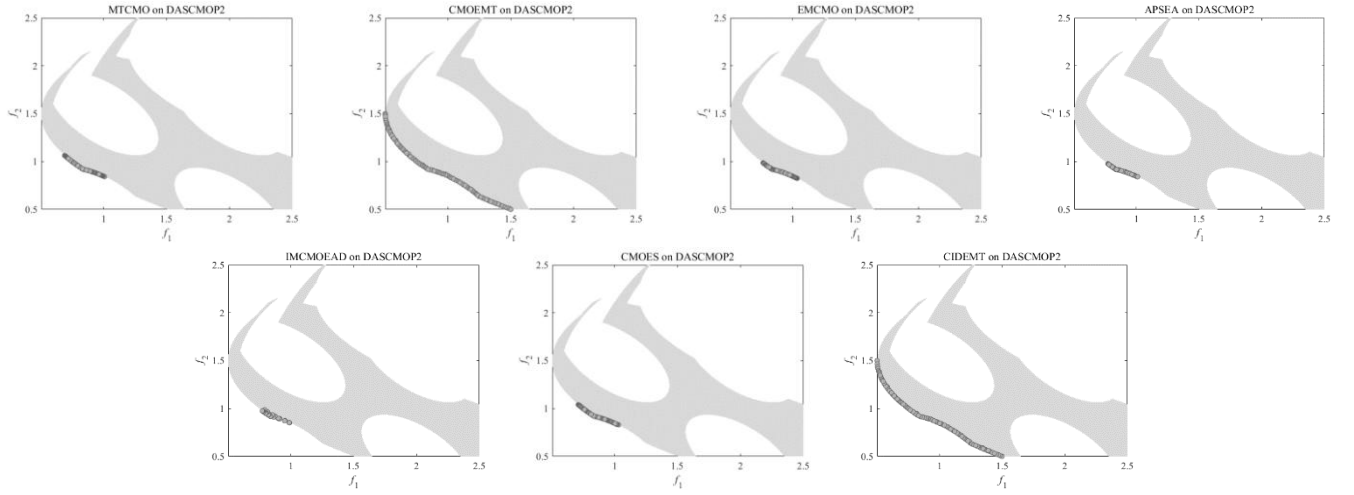


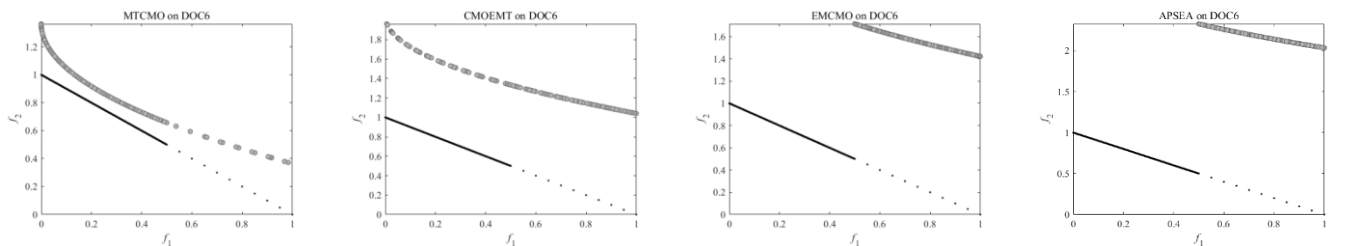
Figure S2 Distribution of final solutions of CIDEMT with 6 benchmark algorithms on DASCMP2

Table S5 Comparison results of IGD values of CIDEMT with 6 benchmark algorithms on DOC

Problem	MTCMO	CMOEMT	EMCMO	APSEA	IMCMOEAD	CMOES	CIDEMT
DOC1	<b>1.1943e-2 (1.37e-2) +</b>	6.0902e-1 (4.96e-1) -	3.4276e+0 (4.75e+0) -	2.6300e+0 (1.88e+0) -	3.3504e+1 (1.15e+1) -	5.5428e-2 (6.04e-2) -	4.4041e-2 (1.52e-1)
DOC2	4.8829e-1 (0.00e+0) =	4.6439e-1 (0.00e+0) =	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	<b>7.6810e-3 (1.62e-3)</b>
DOC3	6.1887e+2 (2.54e+2) -	4.6527e+2 (2.62e+2) -	6.7614e+2 (2.46e+2) -	6.7152e+2 (3.11e+2) -	NaN (NaN)-	6.8219e+2 (1.55e+2) -	<b>8.9826e+1 (1.57e+2)</b>
DOC4	5.5722e-2 (5.98e-2) -	4.6874e-1 (3.36e-1) -	6.3402e-1 (4.46e-1) -	8.6283e-1 (8.90e-1) -	5.8432e-1 (3.23e-1) -	5.1708e-1 (2.47e-1) -	<b>1.5189e-2 (1.89e-3)</b>
DOC5	NaN (NaN)-	3.5446e-1 (1.92e-1) =	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	<b>8.2150e+1 (6.35e+1)</b>
DOC6	6.9037e-1 (9.44e-1) -	7.1650e-1 (4.19e-1) -	2.3306e+0 (2.16e+0) -	1.9092e+0 (2.33e+0) -	3.7713e-1 (2.52e-1) -	2.0492e+0 (1.76e+0) -	<b>2.6822e-3 (8.18e-5)</b>
DOC7	9.8985e-1 (1.47e+0) -	5.7905e-1 (2.59e-1) -	6.3060e+0 (2.40e+0) -	5.2759e+0 (2.21e+0) -	3.1477e+0 (8.70e-1) -	5.6485e+0 (2.03e+0) -	<b>2.4737e-3 (1.12e-4)</b>
DOC8	5.1613e+1 (4.05e+1) -	3.6158e+1 (2.90e+1) -	6.0873e+1 (5.64e+1) -	6.6794e+1 (6.30e+1) -	1.0159e+2 (5.16e+1) -	5.6773e+1 (4.07e+1) -	<b>1.9688e-1 (3.55e-2)</b>
DOC9	1.3592e-1 (9.89e-2) =	1.1232e-1 (8.39e-2) =	1.3695e-1 (8.77e-2) =	1.2585e-1 (9.69e-2) =	8.7341e-1 (1.19e-1) -	1.5660e-1 (9.16e-2) -	<b>5.7741e-2 (3.14e-2)</b>
+/-/=	1/6/2	0/6/3	0/8/1	0/8/1	0/9/0	0/9/0	

Table S6 Comparison results of HV values on DOC between CIDEMT and the 6 benchmark algorithms

Problem	MTCMO	CMOEMT	EMCMO	APSEA	IMCMOEAD	CMOES	CIDEMT
DOC1	3.4095e-1 (1.09e-2) =	9.6334e-2 (1.08e-1) -	2.5443e-2 (6.66e-2) -	2.6273e-2 (7.19e-2) -	0.0000e+0 (0.00e+0) -	3.0581e-1 (4.16e-2) -	<b>3.2984e-1 (5.95e-2)</b>
DOC2	2.1879e-1 (0.00e+0) =	2.6029e-1 (0.00e+0) =	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	<b>6.1562e-1 (2.05e-3)</b>
DOC3	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	NaN (NaN)-	0.0000e+0 (0.00e+0) -	<b>1.7460e-1 (1.32e-1)</b>
DOC4	5.0198e-1 (6.10e-2) -	1.8582e-1 (1.32e-1) -	1.4114e-1 (1.47e-1) -	7.8447e-2 (8.76e-2) -	1.3536e-1 (1.01e-1) -	1.3509e-1 (1.28e-1) -	<b>5.4634e-1 (2.28e-3)</b>
DOC5	NaN (NaN)-	<b>2.3616e-1 (1.68e-1) =</b>	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	NaN (NaN)-	1.3040e-1 (2.01e-1)
DOC6	2.0958e-1 (2.00e-1) -	7.2571e-2 (1.26e-1) -	2.8988e-2 (8.35e-2) -	4.2471e-2 (1.04e-1) -	1.4382e-1 (1.52e-1) -	2.6942e-2 (7.37e-2) -	<b>5.3333e-1 (5.00e-3)</b>
DOC7	1.3571e-1 (1.77e-1) -	5.9272e-2 (9.50e-2) -	0.0000e+0 (0.00e+0) -	3.6895e-3 (2.02e-2) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	<b>5.4462e-1 (5.55e-3)</b>
DOC8	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	<b>6.1663e-1 (5.00e-2)</b>
DOC9	NaN (NaN)=	NaN (NaN)=	NaN (NaN)=	NaN (NaN)=	0.0000e+0 (0.00e+0)=	NaN (NaN)=	NaN (NaN)
+/-/=	0/6/3	0/6/3	0/8/1	0/8/1	0/8/1	0/8/1	



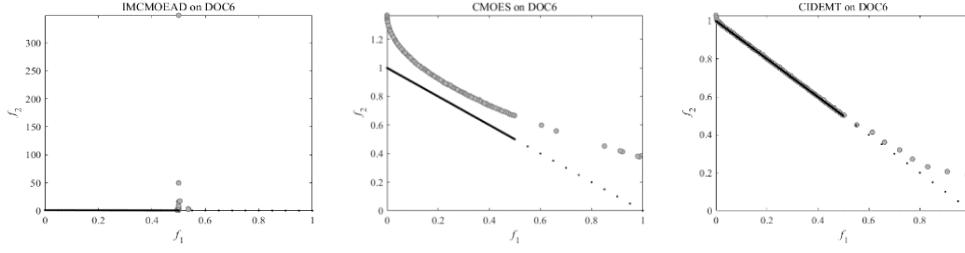


Figure S3 Distribution of final solutions of CIDEMT with 6 benchmark algorithms on DOC6

## 1.2 The visual representations on different types of problems

To more intuitively illustrate the specific performance of the algorithm on different problem types, Figures S4 to S6 present the population distributions of CIDEMT and six baseline algorithms across three types of problems at each evolutionary stage (early, middle, and final). These visualizations demonstrate the detailed behavior of each algorithm across tasks at different stages.

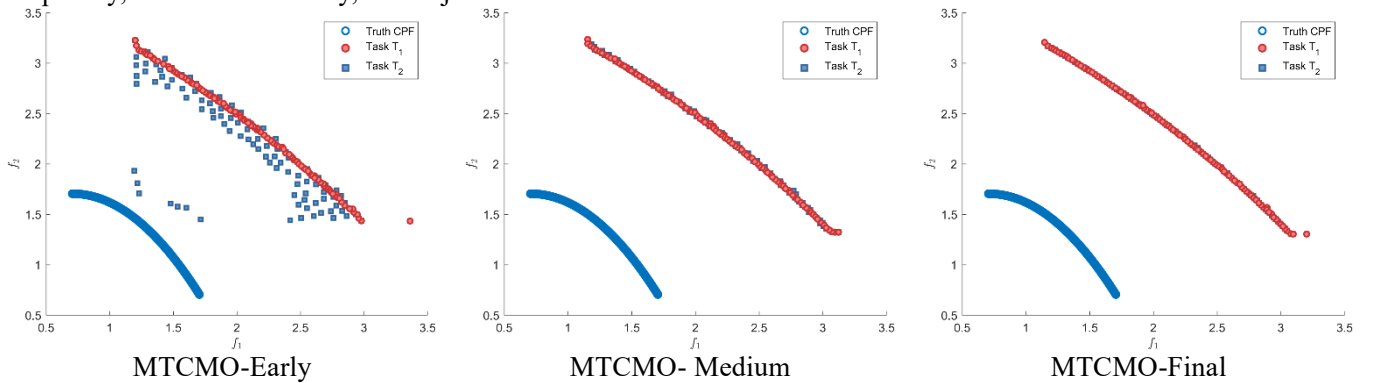
In all three types of multi-objective optimization problems, CIDEMT exhibits stable and powerful adaptability, with particularly outstanding performance in terms of solution distribution and convergence speed.

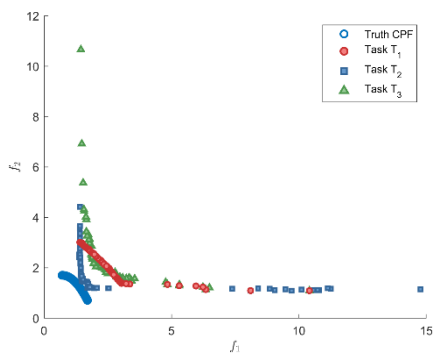
In Type I problems (e.g., LIRCMOP6 in Figure S4), CIDEMT achieves comprehensive task collaboration as early as the initial stage. Task T3 actively explores the boundary region, significantly enhancing solution diversity. During the middle stage, CIDEMT rapidly approaches the true Pareto front. In the final stage, it achieves a high-precision, high-density, and uniformly distributed solution set, clearly outperforming baseline algorithms such as MTCMO and CMOEMT.

In Type II problems (e.g., LIRCMOP11 in Figure S5), CIDEMT can stably generate high-quality feasible solutions under complex constraints. Already in the early stage, the population shows good structural characteristics and distribution trends. In the middle and later stages, feasibility and boundary coverage are continuously improved. By the final stage, the solution sets of all three tasks are uniformly distributed along the true front, significantly outperforming algorithms such as EMCMO and CMOES, which suffer from convergence bias or insufficient task coordination.

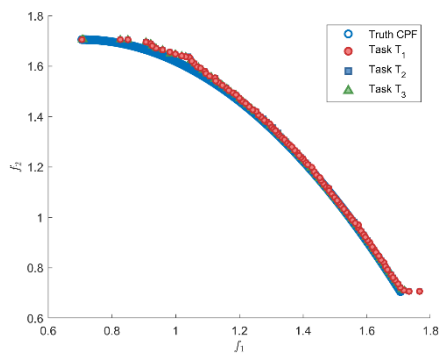
In Type III problems (e.g., LIRCMOP2 in Figure S6), CIDEMT leverages its task complementarity mechanism to effectively traverse narrow feasible regions and multimodal landscapes. Even in the early stage, it achieves broad exploration and coverage. In the middle stage, convergence accelerates. In the final stage, a high-quality solution set closely aligned with the constrained Pareto front (CPF) is obtained. In contrast, other algorithms like IMCMOEAD and APSEA tend to get trapped in local regions, lacking global convergence and distribution balance.

In summary, CIDEMT consistently outperforms the baseline algorithms in terms of solution distribution control and convergence speed. Its multi-task co-evolution mechanism not only accelerates convergence but also enhances the breadth and uniformity of the solution set through effective exploration of boundaries and sparse regions. The consistent performance across all three types of problems validates the efficient integration of its density-guided strategy, boundary exploration capabilities, and constraint-handling methods. These features collectively enable CIDEMT to exhibit strong robustness and generalization ability when addressing multi-objective optimization problems with varying structural complexity, constraint intensity, and objective conflict.

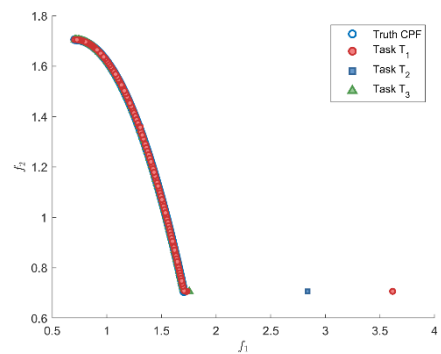




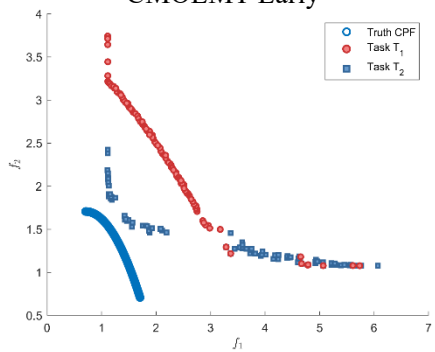
CMOEMT-Early



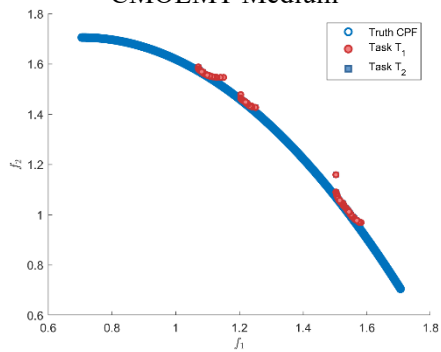
CMOEMT-Medium



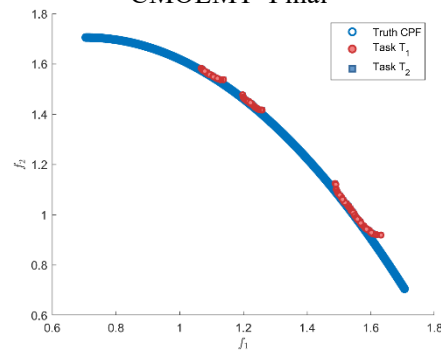
CMOEMT-Final



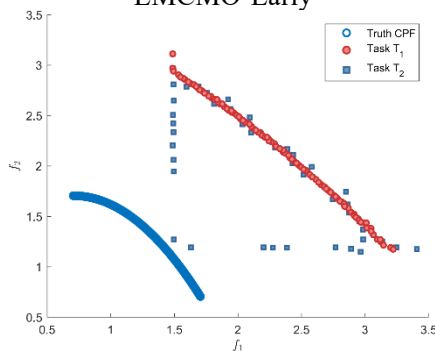
EMCMO-Early



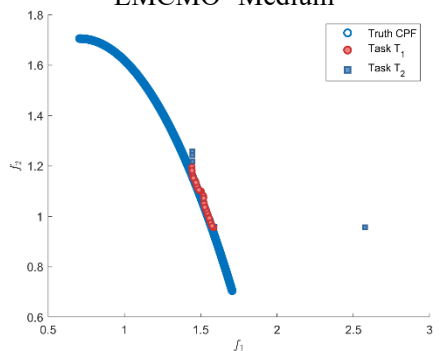
EMCMO-Medium



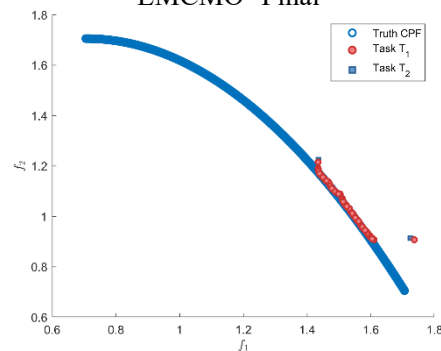
EMCMO-Final



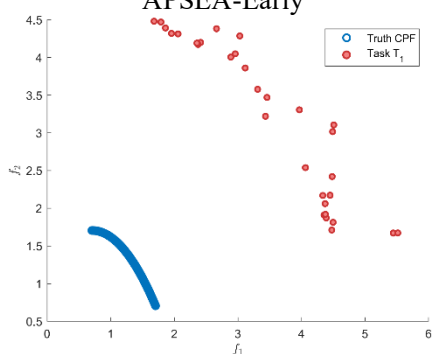
APSEA-Early



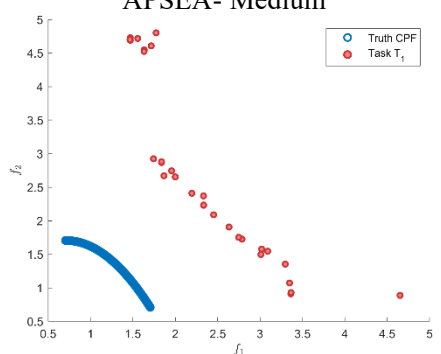
APSEA-Medium



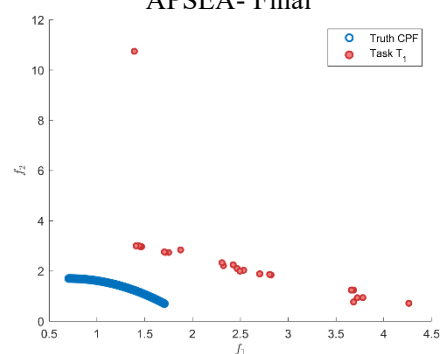
APSEA-Final



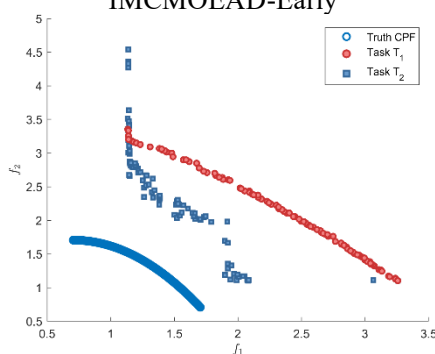
IMCMOEAD-Early



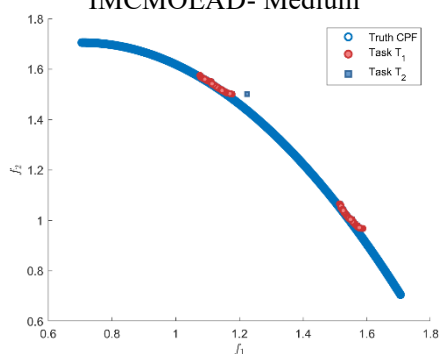
IMCMOEAD-Medium



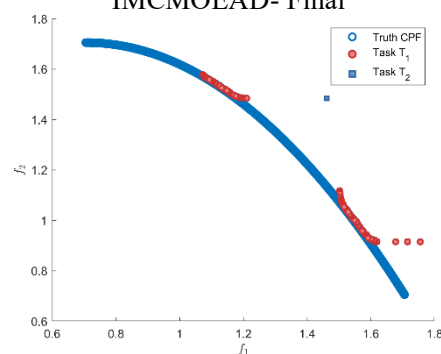
IMCMOEAD-Final



CMOES-Early



CMOES-Medium



CMOES-Final

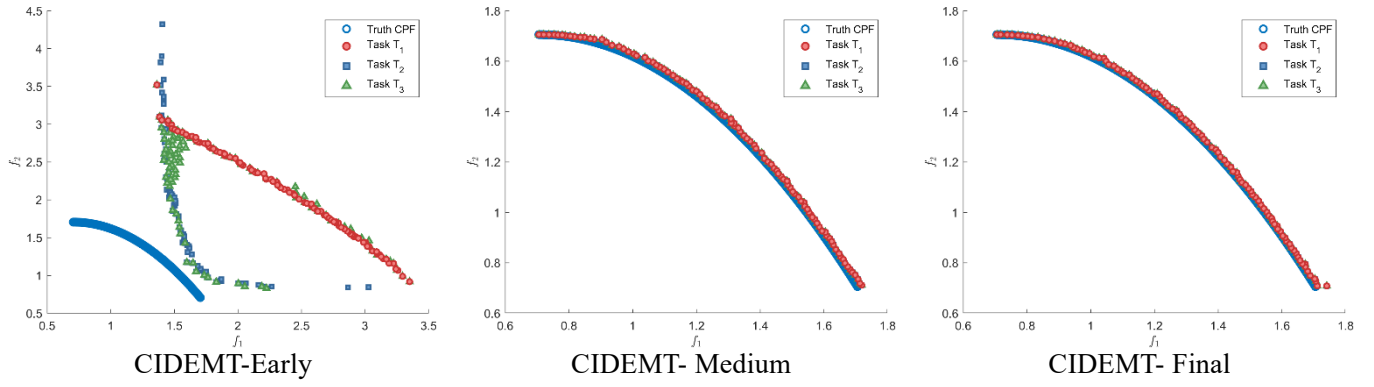
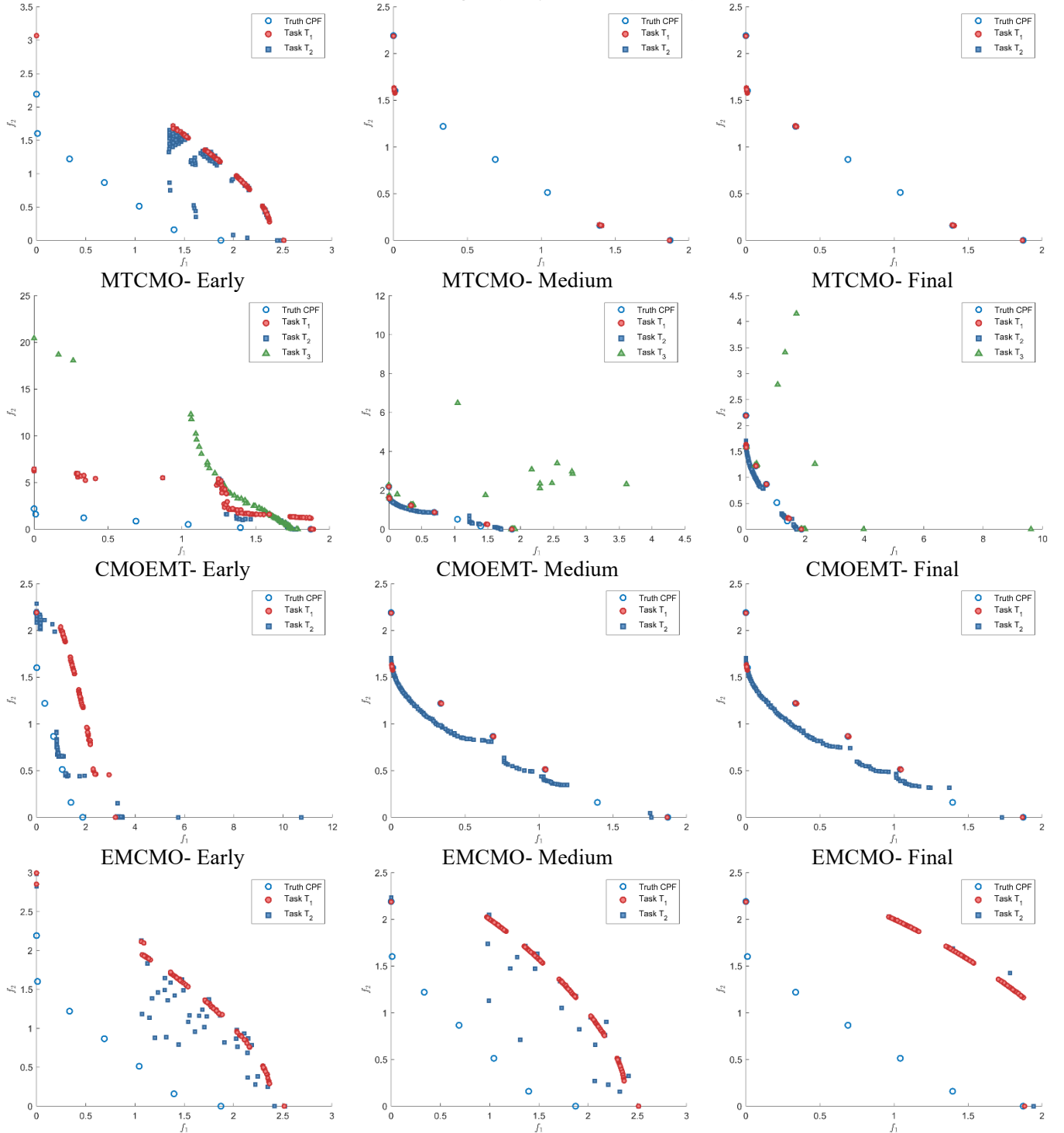


Figure S4 The population distribution of CIDEMENT and 6 benchmark algorithms on the Type I problem LIRCMOP6 at different stages (Early, Medium, Final).



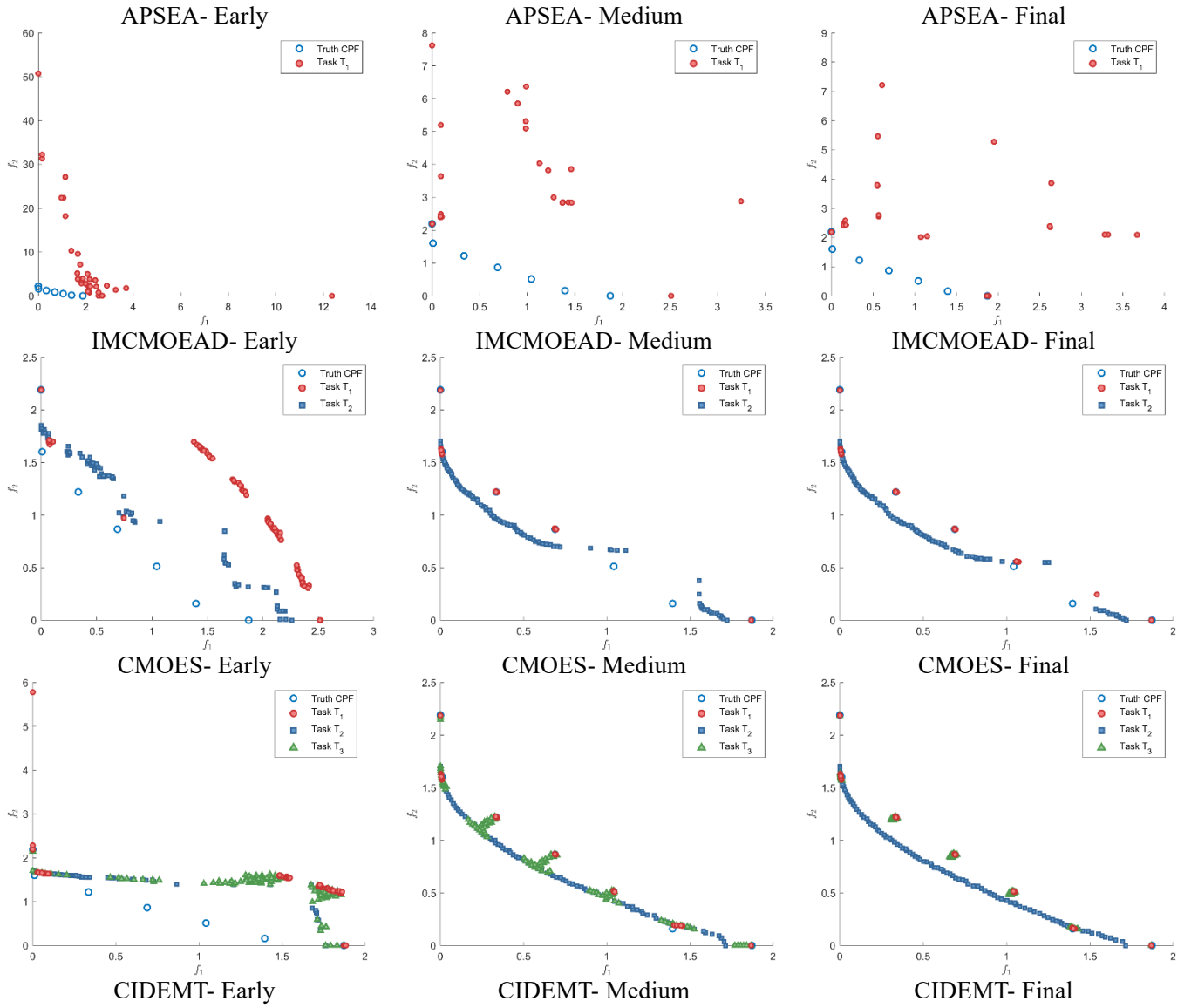
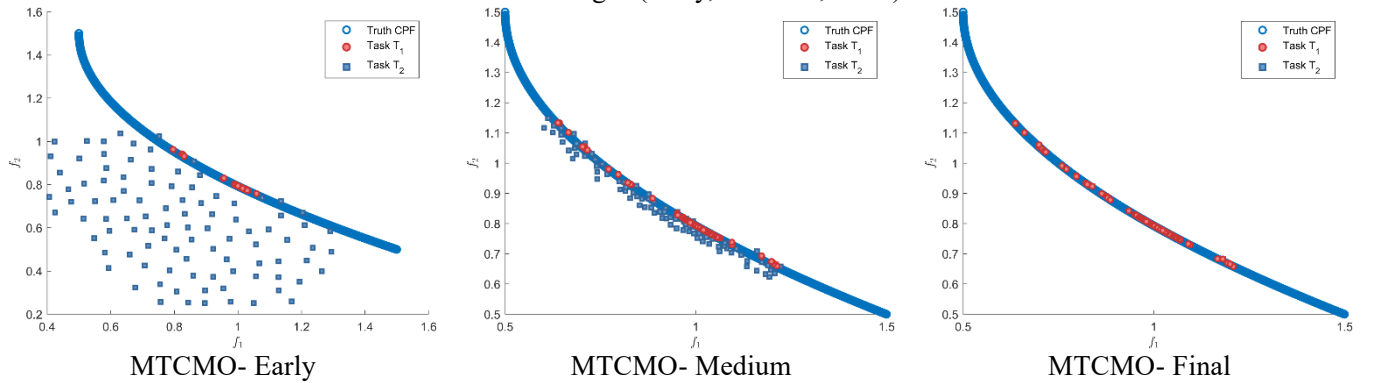
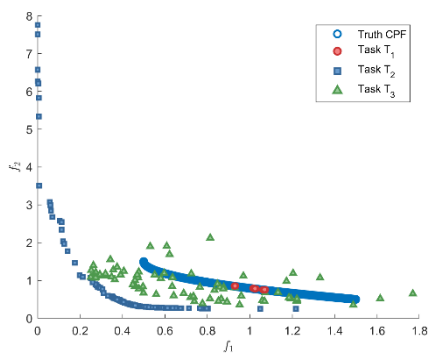


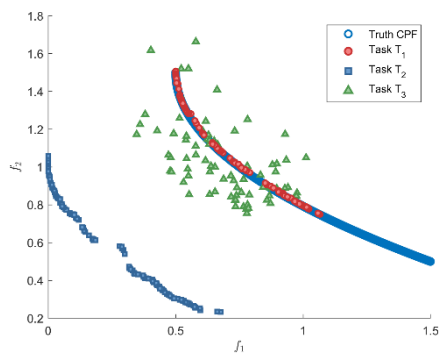
Figure S5 The population distribution of CIDEMT and 6 benchmark algorithms on the Type II problem LIRCMOP11 at different stages (Early, Medium, Final).



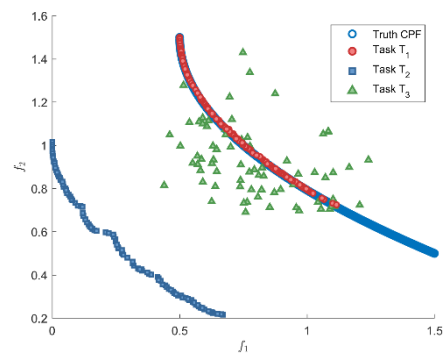




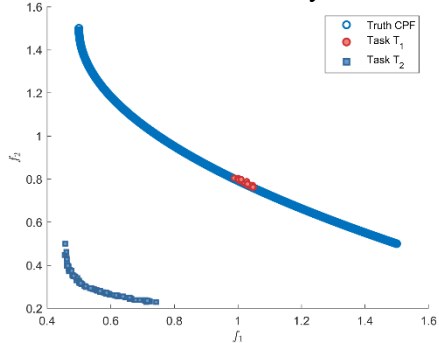
CMOEMT- Early



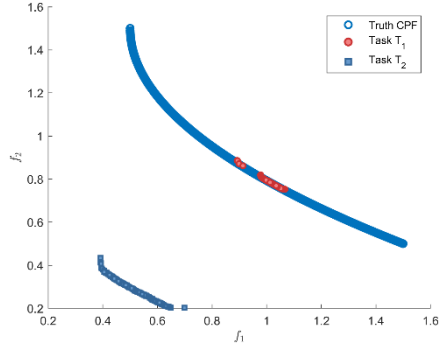
CMOEMT- Medium



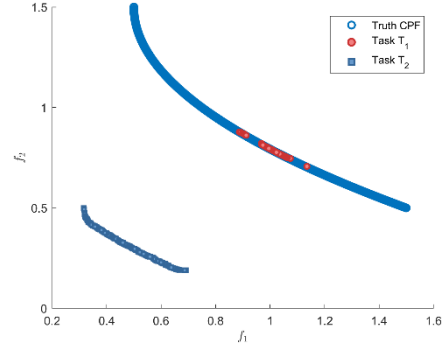
CMOEMT- Final



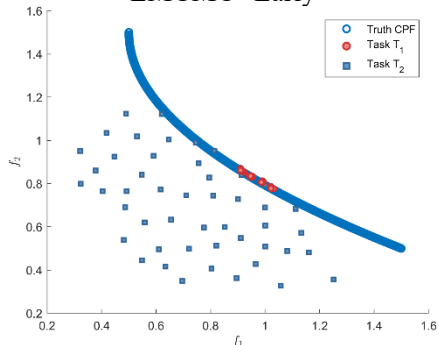
EMCMO- Early



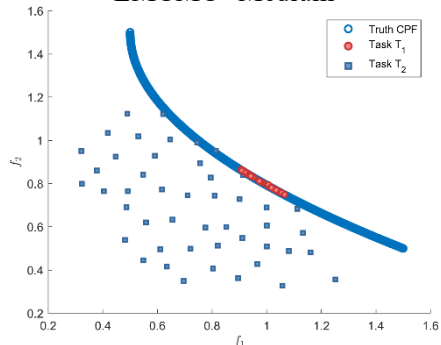
EMCMO- Medium



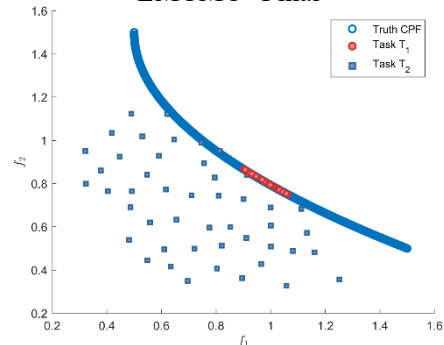
EMCMO- Final



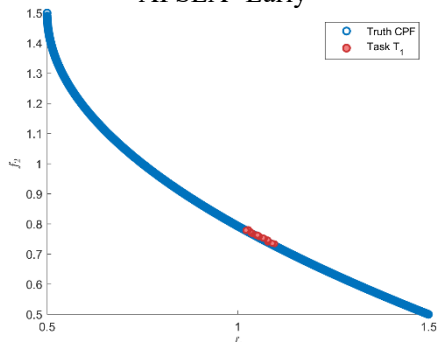
APSEA- Early



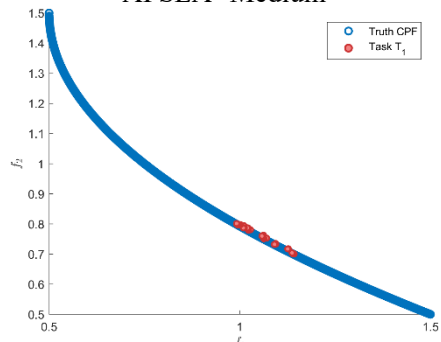
APSEA- Medium



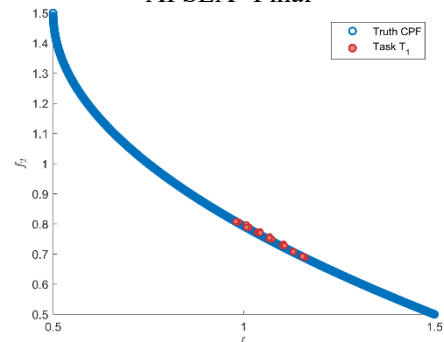
APSEA- Final



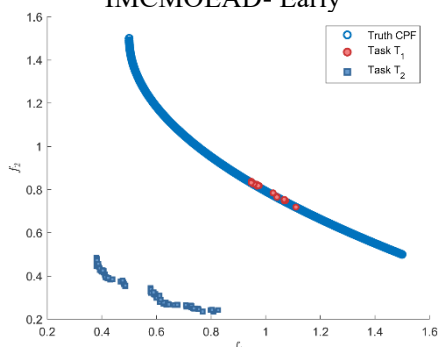
IMCMOEAD- Early



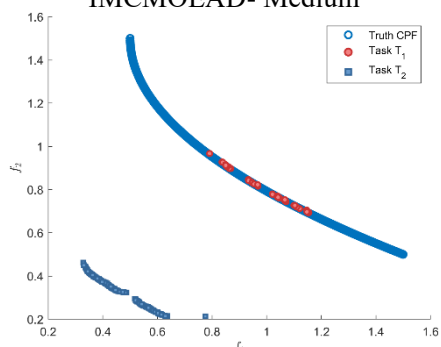
IMCMOEAD- Medium



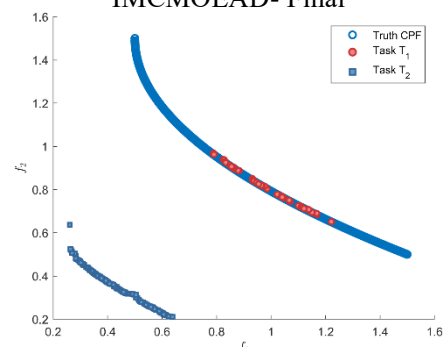
IMCMOEAD- Final



CMOES- Early



CMOES- Medium



CMOES- Final

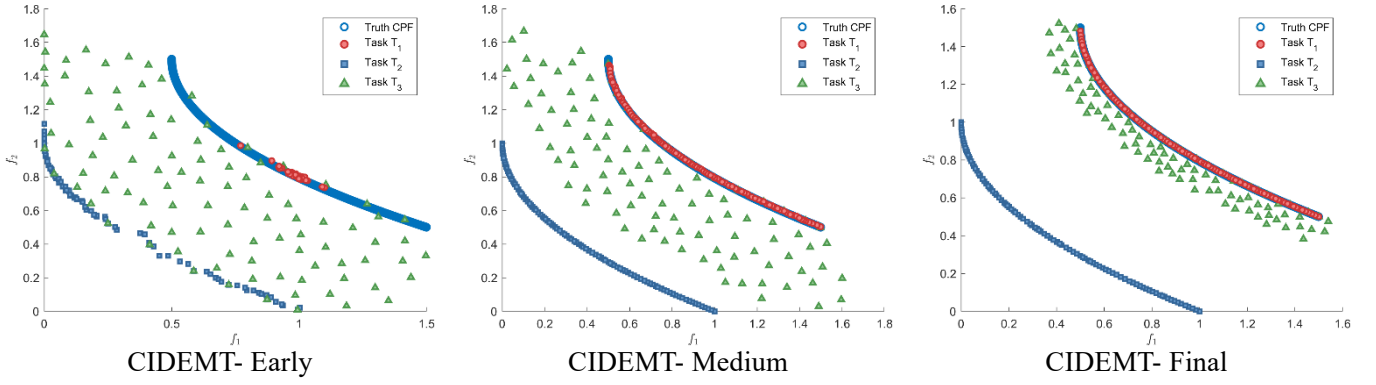


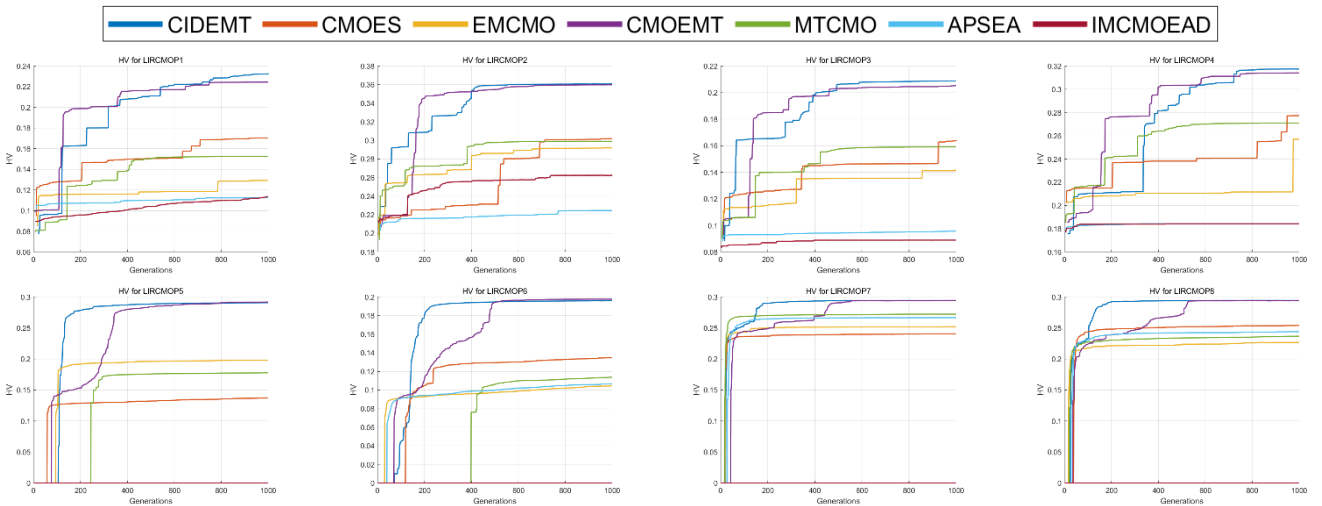
Figure S6 The population distribution of CIDEMT and 6 benchmark algorithms on the Type III problem LIRCMOP2 at different stages (Early, Medium, Final).

### 1.3 Comparison of Convergence Speeds of Different Algorithms

In the comparative study on the LIRCMOP test suite, the CIDEMT algorithm exhibited a marked superiority in convergence speed with respect to both Hypervolume (HV) and Inverted Generational Distance (IGD) metrics. As illustrated in Figure S7, CIDEMT consistently achieved rapid increases in HV within the first few hundred generations across most test functions (e.g., LIRCMOP1–4, 6, 7, 10, and 11), while also maintaining high stability throughout the optimization process. Its final HV values were generally higher than those obtained by the other six benchmark algorithms, indicating significant advantages in both convergence efficiency and solution quality. In contrast, the IMCMOEAD algorithm demonstrated substantial instability, with pronounced fluctuations in multiple test functions—particularly LIRCMOP10 and LIRCMOP11—suggesting poor solution distribution and a lack of reliable convergence behavior.

Figure S8 further substantiates CIDEMT's superiority by highlighting its performance on the IGD metric. CIDEMT consistently exhibited the fastest convergence across nearly all test functions, attaining the lowest final IGD values. This indicates that its generated solution sets are closest to the true Pareto front, reflecting strong performance in multi-objective optimization. While algorithms such as CMOEMT, EMCMO, and CMOES showed competitive performance on certain problems, their overall convergence speed and accuracy lagged behind those of CIDEMT. IMCMOEAD, in particular, suffered from severe performance degradation due to persistent instability and sluggish convergence. In summary, CIDEMT outperformed all competing algorithms in terms of convergence speed, robustness, and solution quality, establishing itself as the most effective approach for the LIRCMOP test problems.

As shown in Table 7, although the time complexity is the same as that of most algorithms, due to the superposition of multiple operations with a time complexity of  $O(NP^2)$ , the running time of CIDEMT is longer compared to the benchmark algorithm. This is also one of the main tasks we will address in the future.



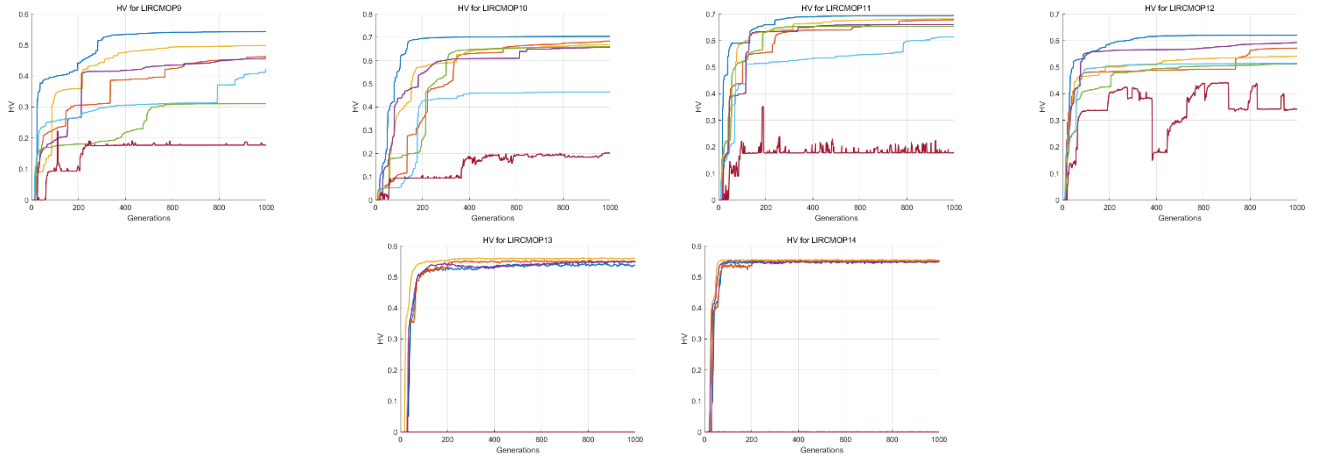


Figure S7 Comparison of HV convergence speeds of CIDEMT and 6 benchmark algorithms on the LIRCMOP test suite.

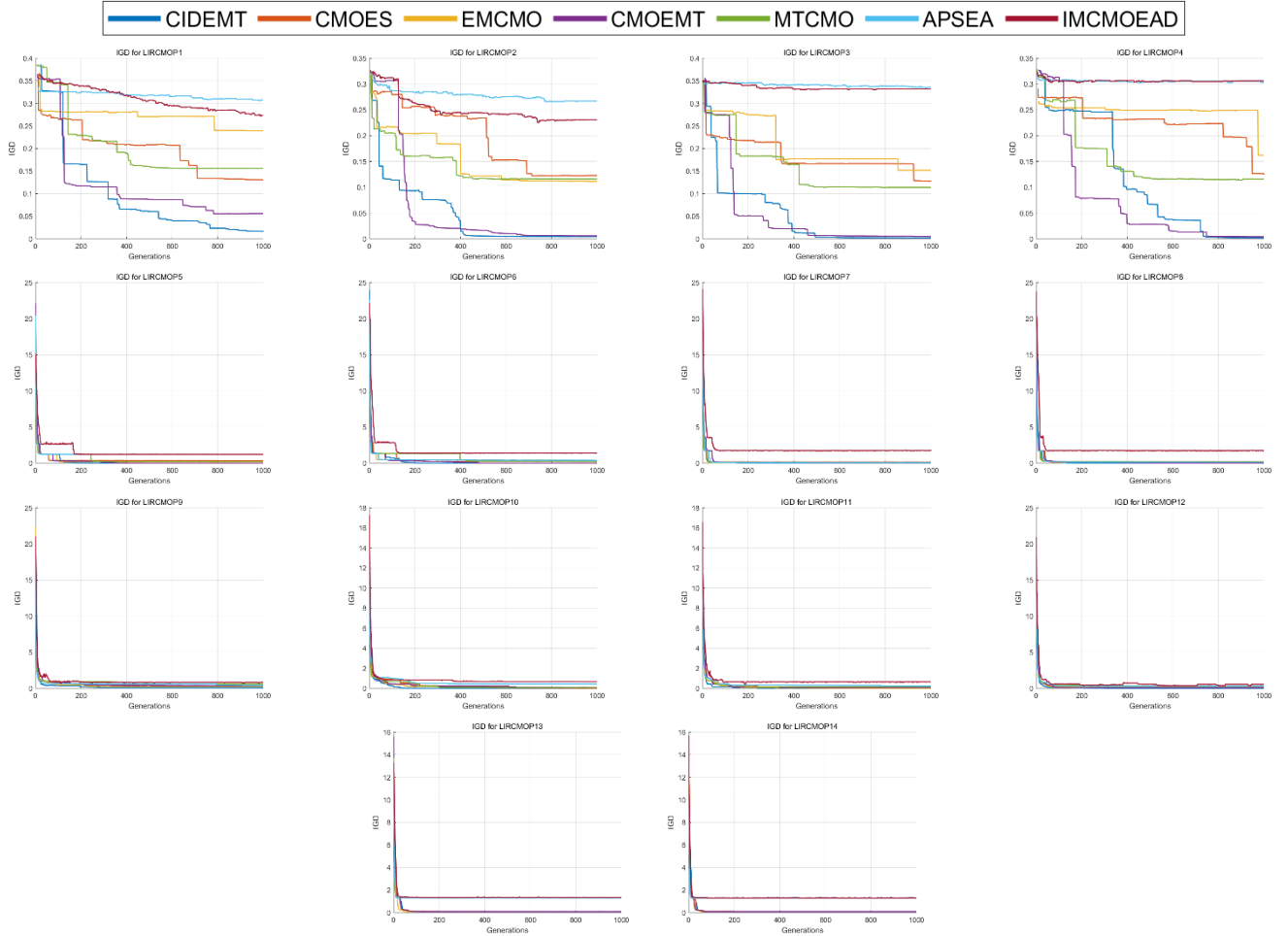


Figure S8 Comparison of IGD convergence speeds of CIDEMT and 6 benchmark algorithms on the LIRCMOP test suite.

Table S7 The average running time of CIDEMT and the benchmark algorithm on LIRCMOP over 30 runs

Problem	CMOES	EMCMO	CMOEMT	MTCMO	APSEA	IMCMOEAD	CIDEMT
LIRCMOP1	3.1700e+2 (3.83e+1)	4.8674e+1 (7.41e-1)	9.4090e+1 (1.17e+0)	6.7467e+1 (4.60e+0)	1.8065e+1 (6.22e-1)	4.8541e+1 (4.40e+0)	2.9834e+2 (1.44e+1)
LIRCMOP2	3.3365e+2 (2.11e+1)	4.8167e+1 (8.47e-1)	9.0936e+1 (1.23e+0)	7.0085e+1 (4.21e+0)	1.7856e+1 (9.80e-1)	4.7450e+1 (4.41e+0)	3.7834e+2 (7.82e+0)
LIRCMOP3	5.1971e+2 (5.61e+1)	4.9050e+1 (1.07e+0)	8.8745e+1 (7.55e-1)	6.8068e+1 (5.61e+0)	1.7579e+1 (7.10e-1)	4.6930e+1 (4.71e+0)	3.4560e+2 (5.75e+0)
LIRCMOP4	5.5245e+2 (4.42e+1)	4.8589e+1 (1.25e+0)	8.9209e+1 (9.97e-1)	7.2238e+1 (3.81e+0)	1.7648e+1 (7.98e-1)	4.5627e+1 (5.07e+0)	3.5878e+2 (1.02e+1)
LIRCMOP5	8.2474e+2 (5.09e+1)	1.7628e+2 (8.77e+0)	5.4299e+2 (2.47e+2)	4.2216e+1 (2.68e+0)	2.4177e+1 (1.73e+0)	4.7634e+1 (8.61e-1)	1.3944e+3 (1.24e+2)
LIRCMOP6	8.1804e+2 (5.73e+1)	1.7772e+2 (8.45e+0)	4.5959e+2 (2.52e+2)	4.4138e+1 (2.49e+0)	2.6059e+1 (1.62e+0)	4.7696e+1 (7.81e-1)	1.4347e+3 (9.97e+0)
LIRCMOP7	1.9887e+2 (1.59e+1)	5.0130e+1 (9.19e-1)	1.0282e+2 (2.26e+0)	6.4311e+1 (1.02e+1)	2.3382e+1 (1.22e+0)	4.8430e+1 (7.69e-1)	2.6221e+2 (7.37e+0)
LIRCMOP8	1.9804e+2 (1.66e+1)	5.0893e+1 (7.66e-1)	1.0542e+2 (2.23e+0)	6.1165e+1 (1.09e+1)	2.3202e+1 (1.59e+0)	4.8537e+1 (8.00e-1)	2.5649e+2 (5.02e+0)
LIRCMOP9	3.7779e+2 (2.81e+1)	1.0111e+2 (1.53e+1)	1.8953e+2 (8.47e+0)	4.6094e+1 (3.31e+0)	2.4786e+1 (2.74e+0)	4.6963e+1 (7.91e-1)	7.4716e+2 (1.29e+1)
LIRCMOP10	5.3342e+2 (4.65e+1)	1.2213e+2 (6.70e+0)	2.9931e+2 (4.51e+1)	5.3873e+1 (3.39e+0)	2.7190e+1 (4.23e+0)	4.7101e+1 (7.22e-1)	1.0143e+3 (7.87e+0)
LIRCMOP11	3.1687e+2 (2.70e+1)	9.3256e+1 (5.34e+0)	1.3770e+2 (8.97e+0)	5.3546e+1 (6.97e+0)	2.3687e+1 (4.40e+0)	4.7255e+1 (6.78e-1)	3.5940e+2 (7.76e+0)
LIRCMOP12	2.5570e+2 (4.51e+1)	5.5674e+1 (1.32e+1)	9.6591e+1 (2.50e+0)	3.9385e+1 (2.24e+0)	2.0784e+1 (1.20e+0)	4.6621e+1 (5.73e-1)	2.7434e+2 (7.83e+0)
LIRCMOP13	2.0353e+3 (8.47e+0)	3.5175e+2 (1.58e+0)	9.0073e+2 (3.01e+1)	7.3110e+1 (2.37e+0)	4.6481e+1 (1.56e+0)	5.4152e+1 (9.00e-1)	2.6290e+3 (1.70e+1)
LIRCMOP14	1.1980e+3 (3.02e+1)	1.5651e+2 (2.13e+0)	5.3239e+2 (2.04e+1)	7.2993e+1 (2.68e+0)	4.6296e+1 (1.12e+0)	5.4772e+1 (2.27e+0)	1.7455e+3 (1.32e+2)

## 2. Exploration of Stage Transition

### 2.1 Analysis of Parameter $\alpha$

The parameter  $\alpha$  in our algorithm is set to a default value of 0.1 based on comprehensive sensitivity analysis. As summarized in Tables S8 and S9, this value demonstrates optimal or near-optimal performance across most of the LIRCMOP test problems. Specifically, in terms of the IGD metric,  $\alpha = 0.1$  yields the best average results among all compared values for 9 out of the 14 problems, while also exhibiting excellent performance according to the HV metric. Compared to alternative values,  $\alpha = 0.1$  most effectively balances the contributions of the CPF and UPF information, without compromising algorithmic robustness. Although stable performance is observed within the range of  $\alpha \in [0.1, 0.5]$ , the consistent superiority of  $\alpha = 0.1$  across the entire problem set supports its selection as the default value, ensuring both general applicability and reliability of the algorithm.

Table S8 IGD Results of Different  $\alpha$  on LIRCMOP

Problem	$\alpha=0$	$\alpha=0.2$	$\alpha=0.5$	$\alpha=1$	$\alpha=0.1$
LIRCMOP1	5.2365e-2 (4.36e-2) -	5.8616e-2 (4.92e-2) -	2.6493e-2 (9.76e-3) -	2.6500e-2 (1.20e-2) -	<b>2.5642e-2 (8.73e-3)</b>
LIRCMOP2	4.2625e-3 (1.86e-4) =	<b>4.2398e-3 (1.53e-4)</b> =	1.3372e-2 (1.25e-2) -	1.5127e-2 (1.34e-2) -	4.2606e-3 (1.66e-4)
LIRCMOP3	3.9878e-2 (5.19e-2) -	3.1073e-2 (4.58e-2) -	2.8048e-3 (2.17e-3) -	2.7260e-3 (1.64e-3) -	<b>2.6198e-3 (1.65e-3)</b>
LIRCMOP4	5.5356e-3 (1.27e-2) -	4.1415e-3 (1.01e-2) -	4.8606e-3 (1.19e-2) -	3.7749e-3 (6.32e-3) -	<b>2.3156e-3 (1.81e-3)</b>
LIRCMOP5	<b>7.0810e-3 (5.78e-4)</b> =	7.3454e-3 (5.95e-4) -	7.2975e-3 (6.28e-4) =	7.2315e-3 (6.11e-4) =	7.1763e-3 (6.92e-4)
LIRCMOP6	7.5220e-3 (2.81e-4) -	<b>7.3888e-3 (3.62e-4)</b> =	7.1940e-3 (4.65e-4) +	7.3686e-3 (3.29e-4) =	7.4105e-3 (4.39e-4)
LIRCMOP7	7.4122e-3 (2.44e-4) -	7.3393e-3 (2.68e-4) =	7.3232e-3 (2.12e-4) -	7.4166e-3 (2.54e-4) -	<b>7.2760e-3 (1.97e-4)</b>
LIRCMOP8	7.3872e-3 (2.06e-4) -	7.3320e-3 (1.92e-4) -	7.2587e-3 (2.00e-4) =	7.3237e-3 (2.47e-4) -	<b>7.2325e-3 (1.93e-4)</b>
LIRCMOP9	<b>2.9652e-2 (2.90e-2)</b> +	7.0531e-2 (2.37e-2) -	5.6139e-2 (3.13e-2) =	6.8811e-2 (2.50e-2) -	6.1985e-2 (2.97e-2)
LIRCMOP10	<b>7.6254e-3 (9.52e-4)</b> +	9.0253e-3 (7.03e-4) -	9.0059e-3 (6.35e-4) -	8.9468e-3 (6.11e-4) -	8.8076e-3 (5.91e-4)
LIRCMOP11	2.7358e-3 (2.22e-4) =	3.0188e-3 (5.13e-4) -	3.4511e-3 (6.16e-4) -	3.2702e-3 (6.10e-4) -	<b>2.6556e-3 (1.54e-4)</b>
LIRCMOP12	4.2952e-3 (6.61e-4) -	3.8536e-3 (3.14e-4) =	3.9019e-3 (4.73e-4) -	6.1784e-3 (1.73e-3) -	<b>3.7282e-3 (3.54e-4)</b>
LIRCMOP13	1.0141e-1 (1.60e-3) -	1.0072e-1 (1.61e-3) =	1.0200e-1 (2.03e-3) -	1.0240e-1 (2.90e-3) -	<b>1.0057e-1 (1.92e-3)</b>
LIRCMOP14	9.7757e-2 (9.62e-4) =	9.7730e-2 (1.09e-3) -	9.7498e-2 (1.01e-3) =	9.7305e-2 (1.02e-3) =	<b>9.7143e-2 (1.31e-3)</b>
+/-/=	2/8/4/	0/9/5	1/9/4	0/11/3	

Table S9 HV Results of Different  $\alpha$  on LIRCMOP

Problem	$\alpha=0$	$\alpha=0.2$	$\alpha=0.5$	$\alpha=1$	$\alpha=0.1$
LIRCMOP1	2.1394e-1 (2.05e-2) -	2.0982e-1 (2.51e-2) -	2.2823e-1 (5.00e-3) -	2.2870e-1 (4.21e-3) -	<b>2.2954e-1 (3.11e-3)</b>
LIRCMOP2	3.6072e-1 (2.11e-4) =	<b>3.6075e-1 (1.80e-4)</b> =	3.5507e-1 (7.79e-3) -	3.5355e-1 (8.94e-3) -	3.6073e-1 (1.49e-4)
LIRCMOP3	1.9320e-1 (2.00e-2) -	1.9608e-1 (1.88e-2) -	2.0805e-1 (1.55e-3) =	2.0811e-1 (1.25e-3) -	<b>2.0818e-1 (1.21e-3)</b>
LIRCMOP4	3.1657e-1 (3.93e-3) -	3.1656e-1 (5.65e-3) -	3.1670e-1 (3.91e-3) -	3.1691e-1 (2.56e-3) -	<b>3.1766e-1 (8.24e-4)</b>
LIRCMOP5	<b>2.9113e-1 (2.71e-4)</b> =	2.9100e-1 (2.71e-4) -	2.9102e-1 (2.94e-4) =	2.9105e-1 (2.89e-4) =	2.9107e-1 (3.20e-4)
LIRCMOP6	1.9594e-1 (1.50e-4) -	1.9600e-1 (1.77e-4) =	<b>1.9610e-1 (2.47e-4)</b> +	1.9601e-1 (1.68e-4) =	1.9600e-1 (2.25e-4)
LIRCMOP7	2.9442e-1 (1.02e-4) -	2.9446e-1 (1.22e-4) =	2.9446e-1 (1.08e-4) -	2.9437e-1 (1.88e-4) -	<b>2.9447e-1 (9.05e-5)</b>
LIRCMOP8	2.9445e-1 (9.49e-5) -	2.9449e-1 (9.07e-5) =	2.9452e-1 (8.49e-5) =	2.9450e-1 (1.14e-4) =	<b>2.9452e-1 (8.88e-5)</b>
LIRCMOP9	<b>5.5652e-1 (7.77e-3)</b> +	5.4531e-1 (5.94e-3) -	5.4885e-1 (8.09e-3) =	5.4574e-1 (6.35e-3) -	5.4768e-1 (7.49e-3)
LIRCMOP10	<b>7.0522e-1 (5.70e-4)</b> +	7.0445e-1 (5.02e-4) -	7.0453e-1 (3.50e-4) =	7.0455e-1 (3.26e-4) -	7.0461e-1 (3.69e-4)
LIRCMOP11	6.9381e-1 (1.31e-4) =	6.9360e-1 (3.51e-4) -	6.9323e-1 (3.65e-4) -	6.9330e-1 (4.93e-4) -	<b>6.9388e-1 (6.65e-5)</b>
LIRCMOP12	6.1986e-1 (2.72e-4) -	6.2006e-1 (1.40e-4) =	6.2005e-1 (1.97e-4) -	6.1873e-1 (7.20e-4) -	<b>6.2012e-1 (1.48e-4)</b>
LIRCMOP13	5.3672e-1 (3.34e-3) -	<b>5.3847e-1 (2.61e-3)</b> =	5.3687e-1 (3.66e-3) -	5.3548e-1 (4.72e-3) -	5.3837e-1 (2.65e-3)
LIRCMOP14	5.4961e-1 (1.73e-3) -	5.4986e-1 (1.92e-3) =	5.5041e-1 (1.76e-3) =	<b>5.5108e-1 (1.37e-3)</b> =	5.5062e-1 (1.63e-3)
+/-/=	2/9/3	0/7/7	1/7/6	0/10/4	

### 2.2 Analysis of Parameter $\lambda$

Based on the experimental results presented in Tables S10 and S11 across 32 benchmark problems, this study systematically evaluates the impact of different  $\lambda$  settings on the performance of the multi-objective evolutionary algorithm CIDEMT. Specifically, comparisons of the IGD and HV performance indicators reveal that setting  $\lambda$  to 1/3 yields consistently superior results across both metrics. CIDEMT achieves the best IGD performance on 17 problems and the best HV performance on 16 problems, indicating its effectiveness in terms of solution convergence, distribution uniformity, and diversity.

In summary, setting  $\lambda$  to 1/3 significantly enhances the overall effectiveness of CIDEMT in solving complex multi-objective optimization problems. This improvement may be attributed to its ability to adaptively select appropriate operators based on problem characteristics, thereby achieving a better balance between search intensification and solution diversity. As a result, the algorithm is less prone to issues such as local optima entrapment or degraded solution distribution.

Table S10 Comparison Results of IGD Values of Different  $\lambda$  in CIDEMT on 32 Test Problems

Problem	$\lambda=1/300$	$\lambda=1/60$	$\lambda=1/30$	$\lambda=1/6$	$\lambda=2/3$	$\lambda=1$	$\lambda=1/3$
LIRCMOP1	3.2858e-2 (1.48e-2) -	3.2051e-2 (1.16e-2) -	3.7156e-2 (2.56e-2) =	4.1421e-2 (3.27e-2) -	3.5117e-2 (1.54e-2) -	3.2573e-2 (1.71e-2) =	<b>2.5642e-2 (8.73e-3)</b>
LIRCMOP2	7.6692e-3 (7.89e-3) -	6.7417e-3 (6.21e-3) -	8.2932e-3 (8.65e-3) -	8.6344e-3 (1.06e-2) -	8.2312e-3 (8.42e-3) -	5.2532e-3 (3.13e-3) -	<b>4.2606e-3 (1.66e-4)</b>
LIRCMOP3	7.1597e-3 (1.86e-2) -	5.9210e-3 (1.18e-2) -	6.9497e-3 (1.35e-2) -	3.4709e-3 (2.33e-3) -	5.5026e-3 (8.36e-3) -	4.4073e-3 (5.92e-3) -	<b>2.6198e-3 (1.65e-3)</b>
LIRCMOP4	1.1738e-2 (2.08e-2) -	1.4002e-2 (2.63e-2) -	1.0387e-2 (2.36e-2) -	9.5195e-3 (1.49e-2) -	8.1178e-3 (1.37e-2) -	1.0183e-2 (1.74e-2) -	<b>2.3156e-3 (1.81e-3)</b>
LIRCMOP5	7.0237e-3 (5.39e-4) =	7.0449e-3 (4.30e-4) =	7.0851e-3 (5.88e-4) =	7.0486e-3 (5.27e-4) =	7.0526e-3 (3.78e-4) =	6.8391e-3 (4.12e-4) =	7.1763e-3 (6.92e-4)

LIRCMOP6	<b>7.3572e-3 (4.32e-4) =</b>	7.4787e-3 (4.05e-4)=	7.4909e-3 (6.01e-4) =	7.4139e-3 (4.69e-4) =	7.4864e-3 (4.05e-4) =	7.4570e-3 (4.34e-4) =	7.4105e-3 (4.39e-4)
LIRCMOP7	7.6248e-3 (3.57e-4) -	7.6496e-3 (3.91e-4) -	7.6066e-3 (2.61e-4) -	7.8946e-3 (7.62e-4) -	7.6307e-3 (3.20e-4) -	7.6237e-3 (3.15e-4) -	<b>7.2760e-3 (1.97e-4)</b>
LIRCMOP8	7.5317e-3 (2.39e-4) -	7.5010e-3 (1.97e-4) -	7.3962e-3 (2.33e-4) -	7.4038e-3 (2.15e-4) -	7.4758e-3 (2.38e-4) -	7.4223e-3 (1.91e-4) -	<b>7.2325e-3 (1.93e-4)</b>
LIRCMOP9	2.3729e-2 (2.31e-2) +	2.5286e-2 (2.26e-2)+	2.2671e-2 (1.99e-2) +	<b>1.5274e-2 (4.41e-3) +</b>	1.8979e-2 (1.75e-2) +	3.0839e-2 (2.85e-2) +	6.1985e-2 (2.97e-2)
LIRCMOP10	8.6493e-3 (5.96e-4) =	8.4775e-3 (7.31e-4)=	<b>8.4124e-3 (7.89e-4) +</b>	8.5485e-3 (6.15e-4) =	8.6407e-3 (8.20e-4) =	8.5590e-3 (4.50e-4) =	8.8076e-3 (5.91e-4)
LIRCMOP11	4.9130e-3 (5.50e-3) -	3.9526e-3 (9.80e-4) -	3.5598e-3 (8.24e-4) -	3.8602e-3 (8.78e-4) -	3.7691e-3 (1.05e-3) -	3.6813e-3 (9.15e-4) -	<b>2.6556e-3 (1.54e-4)</b>
LIRCMOP12	7.0913e-3 (2.64e-3) -	5.8420e-3 (1.71e-3) -	7.1088e-3 (3.64e-3) -	6.1246e-3 (2.19e-3) -	7.8927e-3 (5.03e-3) -	6.3234e-3 (3.42e-3) -	<b>3.7282e-3 (3.54e-4)</b>
LIRCMOP13	1.0114e-1 (1.64e-3) =	1.0071e-1 (1.55e-3)=	1.0132e-1 (2.11e-3) =	1.0110e-1 (1.58e-3) =	1.0104e-1 (1.87e-3) =	1.0104e-1 (1.53e-3) =	<b>1.0057e-1 (1.92e-3)</b>
LIRCMOP14	9.7310e-2 (1.05e-3) =	9.7122e-2 (1.01e-3)=	9.7524e-2 (8.47e-4) =	9.7273e-2 (9.87e-4) =	9.7096e-2 (8.59e-4) =	<b>9.7012e-2 (1.07e-3) =</b>	9.7143e-2 (1.31e-3)
DASCMP1	3.0904e-3 (2.96e-4) -	3.1425e-3 (2.77e-4) -	3.0904e-3 (2.35e-4) -	3.1270e-3 (2.63e-4) -	2.9824e-3 (1.44e-4) -	3.0861e-3 (2.57e-4) -	<b>2.8161e-3 (1.71e-4)</b>
DASCMP2	4.2068e-3 (7.52e-5) =	4.1936e-3 (6.85e-5)=	4.1948e-3 (6.65e-5) +	<b>4.1824e-3 (8.45e-5) +</b>	4.1945e-3 (8.73e-5) =	4.1839e-3 (7.07e-5) +	4.2305e-3 (8.26e-5)
DASCMP3	5.3523e-2 (9.14e-2) -	1.1341e-1 (1.19e-1) -	6.6970e-2 (9.70e-2) -	5.8346e-2 (9.00e-2) -	4.7069e-2 (7.53e-2) -	5.0417e-2 (8.22e-2) -	<b>1.4085e-2 (2.36e-3)</b>
DASCMP4	1.3609e-3 (7.36e-4) -	1.1707e-3 (4.54e-5)+	1.2522e-3 (5.21e-4) +	<b>1.1631e-3 (3.72e-5) +</b>	1.2700e-3 (5.23e-4) -	1.2685e-3 (5.20e-4) -	1.2659e-3 (9.80e-5)
DASCMP5	2.7823e-3 (7.84e-5) =	2.7714e-3 (1.09e-4)+	2.7798e-3 (7.84e-5) =	<b>2.7620e-3 (6.57e-5) +</b>	2.7811e-3 (1.06e-4) =	2.7702e-3 (6.32e-5) =	2.8016e-3 (7.10e-5)
DASCMP6	1.8509e-2 (2.38e-3) -	1.8315e-2 (2.61e-3) -	1.8793e-2 (2.12e-3) -	1.8744e-2 (2.07e-3) -	1.9008e-2 (1.76e-3) -	1.8322e-2 (2.59e-3) -	<b>1.4151e-2 (2.95e-3)</b>
DASCMP7	3.1122e-2 (6.36e-4) +	3.1052e-2 (5.13e-4)+	3.0979e-2 (4.88e-4) +	<b>3.0922e-2 (6.27e-4) +</b>	3.1158e-2 (8.74e-4) +	3.1313e-2 (1.06e-3) +	3.1791e-2 (6.67e-4)
DASCMP8	4.0243e-2 (7.38e-4) =	3.9979e-2 (8.70e-4)=	4.0396e-2 (9.31e-4) -	4.0220e-2 (1.03e-3) =	3.9965e-2 (7.77e-4) =	3.9842e-2 (1.10e-3) =	<b>3.9838e-2 (8.74e-4)</b>
DASCMP9	3.9520e-2 (8.86e-4) =	3.9594e-2 (9.68e-4)=	3.9715e-2 (1.02e-3) =	3.9497e-2 (1.00e-3) =	3.9605e-2 (1.08e-3) =	3.9651e-2 (7.53e-4) =	<b>3.9325e-2 (8.57e-4)</b>
DOC1	5.7798e-3 (4.02e-3) +	2.6465e-2 (1.18e-1)=	1.8298e-2 (7.21e-2) =	5.2070e-3 (7.55e-4) =	<b>5.1607e-3 (5.17e-4) =</b>	2.7985e-2 (1.25e-1) =	4.4041e-2 (1.52e-1)
DOC2	8.8752e-3 (2.78e-3) =	9.3892e-3 (3.95e-3)=	8.3247e-3 (1.87e-3) =	9.0306e-3 (2.34e-3) -	8.1958e-3 (2.16e-3) =	7.8257e-3 (1.63e-3) =	<b>7.6810e-3 (1.62e-3)</b>
DOC3	9.2573e+1 (1.21e+2) =	6.3447e+1(1.14e+2)=	1.1571e+2 (1.60e+2) -	7.6928e+1 (1.10e+2) =	<b>5.3248e+1 (9.39e+1) =</b>	1.0608e+2 (1.45e+2) =	8.9826e+1 (1.57e+2)
DOC4	1.7497e-2 (3.80e-3) -	1.9616e-2 (5.58e-3) -	1.8735e-2 (4.79e-3) -	1.7394e-2 (4.63e-3) =	1.8664e-2 (6.95e-3) -	1.9298e-2 (8.12e-3) =	<b>1.5189e-2 (1.89e-3)</b>
DOC5	5.1711e+1 (6.00e+1) +	7.1758e+1(6.02e+1)=	<b>4.7420e+1 (6.25e+1) +</b>	5.3654e+1 (6.42e+1) +	8.0308e+1 (5.73e+1) =	7.1057e+1 (5.87e+1) =	8.2150e+1 (6.35e+1)
DOC6	2.6727e-3 (1.75e-4) =	<b>2.6393e-3 (1.16e-4) +</b>	1.1978e-2 (5.11e-2) =	2.7624e-3 (2.59e-4) =	2.6520e-3 (1.24e-4) =	2.6530e-3 (9.71e-5) =	2.6822e-3 (8.18e-5)
DOC7	9.6108e-1 (1.01e+0) -	4.3905e-1 (6.31e-1) -	7.9456e-1 (1.04e+0) -	6.2190e-1 (6.65e-1) -	7.8265e-1 (9.86e-1) -	7.1539e-1 (9.80e-1) -	<b>2.4737e-3 (1.12e-4)</b>
DOC8	1.5535e-1 (4.27e-2) +	1.4769e-1 (2.49e-2) +	1.4782e-1 (2.25e-2) +	<b>1.4232e-1 (1.50e-2) +</b>	1.4273e-1 (2.03e-2) +	1.4491e-1 (1.75e-2) +	1.9688e-1 (3.55e-2)
DOC9	5.1488e-2 (3.08e-2) +	5.1606e-2 (3.16e-2) +	5.1066e-2 (3.08e-2) +	<b>4.6594e-2 (6.97e-3) +</b>	5.2909e-2 (3.08e-2) +	4.7236e-2 (7.04e-3) +	5.7741e-2 (3.14e-2)
+/-/=	6/14/12	7/13/12	8/14/10	8/13/11	4/14/14	5/12/15	

Table S11 Comparison Results of HV Values of Different  $\lambda$  in 32 Test Problems of CIDEMT

Problem	$\lambda=1/300$	$\lambda=1/60$	$\lambda=1/30$	$\lambda=1/6$	$\lambda=2/3$	$\lambda=1$	$\lambda=1/3$
LIRCMOP1	2.2320e-1 (7.99e-3) -	2.2359e-1 (6.01e-3) -	2.2100e-1 (1.21e-2) -	2.1813e-1 (1.77e-2) -	2.2379e-1 (6.60e-3) -	2.2488e-1 (8.29e-3) -	<b>2.2954e-1 (3.11e-3)</b>
LIRCMOP2	3.5847e-1 (4.50e-3) -	3.5902e-1 (3.73e-3) -	3.5791e-1 (5.26e-3) -	3.5806e-1 (5.82e-3) -	3.5797e-1 (5.65e-3) -	3.5989e-1 (1.90e-3) -	<b>3.6073e-1 (1.49e-4)</b>
LIRCMOP3	2.0603e-1 (7.87e-3) -	2.0656e-1 (4.14e-3) -	2.0579e-1 (6.19e-3) -	2.0740e-1 (1.63e-3) -	2.0644e-1 (3.61e-3) -	2.0719e-1 (2.96e-3) -	<b>2.0818e-1 (1.21e-3)</b>
LIRCMOP4	3.1325e-1 (8.16e-3) -	3.1136e-1 (1.07e-2) -	3.1292e-1 (1.15e-2) -	3.1425e-1 (5.04e-3) -	3.1444e-1 (4.56e-3) -	3.1309e-1 (8.16e-3) -	<b>3.1766e-1 (8.24e-4)</b>
LIRCMOP5	2.9114e-1 (2.66e-4) =	2.9112e-1 (1.88e-4) =	2.9112e-1 (2.67e-4) =	2.9113e-1 (2.52e-4) =	2.9112e-1 (1.76e-4) =	<b>2.9122e-1 (1.84e-4) =</b>	2.9107e-1 (3.20e-4)
LIRCMOP6	<b>1.9601e-1 (2.28e-4) =</b>	1.9596e-1 (2.00e-4) =	1.9593e-1 (3.01e-4) =	1.9599e-1 (2.43e-4) =	1.9591e-1 (2.15e-4) =	1.9594e-1 (2.40e-4) =	1.9600e-1 (2.25e-4)
LIRCMOP7	2.9407e-1 (2.65e-4) -	2.9409e-1 (3.21e-4) -	2.9419e-1 (2.38e-4) -	2.9391e-1 (6.53e-4) -	2.9413e-1 (2.28e-4) -	2.9407e-1 (2.76e-4) -	<b>2.9447e-1 (9.05e-5)</b>
LIRCMOP8	2.9440e-1 (1.20e-4) -	2.9441e-1 (1.10e-4) -	2.9446e-1 (1.27e-4) =	2.9446e-1 (9.30e-5) =	2.9443e-1 (1.18e-4) -	2.9445e-1 (8.92e-5) -	<b>2.9452e-1 (8.88e-5)</b>
LIRCMOP9	5.5765e-1 (6.05e-3) +	5.5703e-1 (5.65e-3) +	5.5768e-1 (5.01e-3) +	<b>5.5973e-1 (1.99e-3) +</b>	5.5898e-1 (4.96e-3) +	5.5573e-1 (7.59e-3) +	5.4768e-1 (7.49e-3)
LIRCMOP10	7.0465e-1 (4.41e-4) =	7.0477e-1 (4.21e-4) =	<b>7.0482e-1 (4.53e-4) =</b>	7.0473e-1 (4.24e-4) =	7.0463e-1 (4.75e-4) =	7.0476e-1 (3.19e-4) =	7.0461e-1 (3.69e-4)
LIRCMOP11	6.9223e-1 (3.61e-3) -	6.9284e-1 (8.71e-4) -	6.9310e-1 (6.71e-4) -	6.9288e-1 (7.65e-4) -	6.9297e-1 (6.93e-4) -	6.9306e-1 (6.15e-4) -	<b>6.9388e-1 (6.65e-5)</b>
LIRCMOP12	6.1831e-1 (1.36e-3) -	6.1895e-1 (9.66e-4) -	6.1823e-1 (1.94e-3) -	6.1882e-1 (1.09e-3) -	6.1789e-1 (2.52e-3) -	6.1864e-1 (1.67e-3) -	<b>6.2012e-1 (1.48e-4)</b>
LIRCMOP13	5.3821e-1 (2.38e-3) =	<b>5.3919e-1 (2.69e-3) =</b>	5.3860e-1 (2.73e-3) =	5.3830e-1 (2.78e-3) =	5.3892e-1 (2.85e-3) =	5.3810e-1 (2.38e-3) =	5.3837e-1 (2.65e-3)
LIRCMOP14	5.5098e-1 (1.46e-3) =	5.5113e-1 (1.63e-3) =	5.5060e-1 (1.51e-3) =	5.5105e-1 (1.34e-3) =	5.5104e-1 (1.65e-3) =	<b>5.5139e-1 (1.58e-3) =</b>	5.5062e-1 (1.63e-3)
DASCMP1	2.1235e-1 (3.57e-4) -	2.1249e-1 (3.44e-4) -	2.1235e-1 (4.42e-4) -	2.1233e-1 (3.96e-4) -	2.1239e-1 (4.11e-4) -	2.1223e-1 (3.48e-4) -	<b>2.1277e-1 (1.86e-4)</b>

DASCMOP2	3.5519e-1 (1.52e-4) -	3.5517e-1 (1.61e-4) -	3.5524e-1 (1.01e-4) =	3.5525e-1 (1.04e-4) =	3.5520e-1 (1.33e-4) -	3.5518e-1 (1.12e-4) -	<b>3.5529e-1 (6.42e-5)</b>
DASCMOP3	2.9911e-1 (3.34e-2) -	2.7495e-1 (4.65e-2) -	2.9404e-1 (3.69e-2) -	2.9608e-1 (3.62e-2) -	3.0009e-1 (3.10e-2) -	2.9924e-1 (3.30e-2) -	<b>3.1232e-1 (1.31e-4)</b>
DASCMOP4	2.0363e-1 (2.14e-3) -	2.0418e-1 (2.67e-4) +	2.0396e-1 (1.54e-3) -	<b>2.0418e-1 (3.16e-4) +</b>	2.0392e-1 (1.53e-3) -	2.0384e-1 (1.55e-3) -	2.0410e-1 (1.78e-4)
DASCMOP5	3.5150e-1 (1.35e-4) +	<b>3.5153e-1 (1.59e-4) +</b>	3.5148e-1 (1.46e-4) =	3.5149e-1 (1.46e-4) +	3.5148e-1 (1.25e-4) +	3.5151e-1 (1.57e-4) +	3.5142e-1 (1.11e-4)
DASCMOP6	3.1240e-1 (1.01e-4) -	3.1243e-1 (9.38e-5) =	3.1235e-1 (1.92e-4) -	3.1237e-1 (1.93e-4) -	3.1237e-1 (1.23e-4) -	3.1240e-1 (1.29e-4) -	<b>3.1247e-1 (7.43e-5)</b>
DASCMOP7	2.8806e-1 (5.34e-4) +	2.8807e-1 (5.27e-4) +	2.8811e-1 (5.81e-4) +	<b>2.8821e-1 (4.12e-4) +</b>	2.8793e-1 (4.91e-4) +	2.8784e-1 (5.21e-4) +	2.8748e-1 (3.98e-4)
DASCMOP8	2.0662e-1 (5.36e-4) +	2.0671e-1 (5.30e-4) +	2.0671e-1 (5.00e-4) +	2.0662e-1 (5.62e-4) +	2.0672e-1 (4.17e-4) +	<b>2.0684e-1 (4.65e-4) +</b>	2.0596e-1 (4.18e-4)
DASCMOP9	2.0621e-1 (3.59e-4) =	2.0617e-1 (3.62e-4) =	2.0610e-1 (3.37e-4) =	2.0613e-1 (4.15e-4) =	<b>2.0623e-1 (2.91e-4) =</b>	2.0621e-1 (3.74e-4) =	2.0621e-1 (3.42e-4)
DOC1	3.4432e-1 (3.52e-3) +	3.3714e-1 (4.56e-2) =	3.3628e-1 (4.94e-2) =	3.4535e-1 (8.39e-4) =	<b>3.4549e-1 (8.34e-4) =</b>	3.3698e-1 (4.62e-2) =	3.2984e-1 (5.95e-2)
DOC2	6.1380e-1 (3.80e-3) =	6.1326e-1 (5.11e-3) =	6.1472e-1 (2.46e-3) =	6.1347e-1 (3.23e-3) -	6.1463e-1 (3.04e-3) =	6.1522e-1 (2.12e-3) =	<b>6.1562e-1 (2.05e-3)</b>
DOC3	7.7745e-2 (1.32e-1) -	1.4563e-1 (1.52e-1) =	4.6368e-2 (1.06e-1) -	9.9566e-2 (1.42e-1) =	1.2268e-1 (1.51e-1) =	7.2970e-2 (1.35e-1) -	<b>1.7460e-1 (1.32e-1)</b>
DOC4	5.4351e-1 (4.43e-3) -	5.4104e-1 (6.35e-3) -	5.4206e-1 (5.55e-3) -	5.4365e-1 (5.46e-3) -	5.4217e-1 (7.94e-3) -	5.4153e-1 (9.35e-3) =	<b>5.4634e-1 (2.28e-3)</b>
DOC5	2.5859e-1 (2.40e-1) +	1.5167e-1 (2.10e-1) =	<b>2.7107e-1 (2.26e-1) +</b>	2.5503e-1 (2.31e-1) =	1.2997e-1 (2.13e-1) =	1.7181e-1 (2.35e-1) =	1.3040e-1 (2.01e-1)
DOC6	5.4088e-1 (5.23e-3) +	5.3839e-1 (5.23e-3) +	5.2725e-1 (6.50e-2) -	5.3749e-1 (6.56e-3) +	<b>5.4096e-1 (5.32e-3) +</b>	5.3902e-1 (5.65e-3) +	5.3333e-1 (5.00e-3)
DOC7	1.8970e-1 (2.41e-1) -	3.2477e-1 (2.53e-1) -	3.0109e-1 (2.60e-1) -	2.2668e-1 (2.53e-1) -	2.5377e-1 (2.39e-1) -	2.9789e-1 (2.64e-1) -	<b>5.4462e-1 (5.55e-3)</b>
DOC8	6.7424e-1 (5.90e-2) +	6.8421e-1 (3.24e-2) +	6.8504e-1 (3.12e-2) +	<b>6.9115e-1 (2.03e-2) +</b>	6.9101e-1 (2.71e-2) +	6.8793e-1 (2.53e-2) +	6.1663e-1 (5.00e-2)
DOC9	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)
+/-/=	8/16/7	7/13/11	5/15/11	7/13/11	6/15/10	6/15/10	

## 2.2 The investigation of two stages switching methods

In this paper, the switching conditions between the two stages are controlled by a fixed parameter ( $\lambda$ ). The aim is to investigate the impact of switching between these stages on the algorithm's performance. To this end, the adaptive switching method proposed in reference [1] is integrated into CIDEMT, resulting in a new method named S-CIDEMT. This adaptive switching method detects whether the population has converged to the optimal feasible solution (UPF). If, during generation  $g$ , the average value of each objective function changes less than a predefined threshold  $\theta$ , the first stage is considered complete, and the next stage begins. This threshold controls the convergence behavior of the algorithm in the first stage: when  $\theta$  is set to a larger value, the switching process is activated even if the population has not fully converged to the UPF; conversely, a smaller  $\theta$  value leads to a more precise convergence. Based on this adaptive method, four different values of  $\theta$  (0.5, 0.1, 0.001, 0.0001) are tested, and the corresponding algorithms are named S-CIDEMT-1, S-CIDEMT-2, S-CIDEMT-3, and S-CIDEMT-4. The comparison results on the LIRCMOP test suite, shown in Tables S12 and S13, indicate that as  $\theta$  decreases from 0.5 to 0.01, the number of instances where the comparison method performs worse than CIDEMT decreases. However, when  $\theta$  is further reduced from 0.01 to 0.0001, the number of such instances increases. This suggests that when  $\theta$  is large (e.g., 0.5), the convergence performance of the second population is weaker, which negatively affects the correct identification of the problem type. As  $\theta$  decreases, the second population converges more effectively on the UPF, as more function evaluations are allocated to the first stage. Clearly, the results of these four variants show that both extreme cases—over-searching and insufficient search in the first stage—can still yield very good results. This indicates that it is not necessary for the two populations to be very close to the CPF and UPF in the first stage, as long as the problem type can be probabilistically estimated accurately with minimal resources. Furthermore, CIDEMT outperforms all four variants, possibly due to the current inefficiency of the adaptive mechanism, suggesting that further improvements in the switching method are needed. Therefore, exploring more efficient switching strategies is a worthwhile avenue for future research.

Table S12 Adaptive stage transition and the CIDEMT algorithm's IGD results on different types of problems

Type	Problem	S- CIDEMT-1	S- CIDEMT-2	S- CIDEMT-3	S- CIDEMT-4	CIDEMT
I	LIRCMOP5	7.2182e-3 (5.62e-4) =	7.0970e-3 (5.56e-4) =	<b>7.0527e-3 (4.62e-4) =</b>	7.0904e-3 (4.73e-4) =	7.1763e-3 (6.92e-4)
	LIRCMOP6	<b>7.3928e-3 (3.96e-4) =</b>	7.4644e-3 (4.11e-4) =	7.4331e-3 (3.17e-4) =	7.4628e-3 (4.57e-4) =	7.4105e-3 (4.39e-4)
	LIRCMOP9	2.2688e-2 (2.38e-2) +	3.2456e-2 (3.02e-2) +	<b>1.9179e-2 (1.72e-2) +</b>	2.9100e-2 (2.91e-2) +	6.1985e-2 (2.97e-2)
	LIRCMOP10	9.0337e-3 (6.09e-4) +	7.4370e-3 (6.99e-4) +	<b>7.1896e-3 (8.94e-4) +</b>	7.5927e-3 (1.03e-3) +	8.8076e-3 (5.91e-4)
	LIRCMOP13	1.0224e-1 (2.78e-3) -	1.0252e-1 (2.51e-3) -	1.0214e-1 (2.29e-3) -	1.0264e-1 (2.21e-3) -	<b>1.0057e-1 (1.92e-3)</b>
II	LIRCMOP14	9.7541e-2 (1.37e-3) =	9.7320e-2 (1.15e-3) =	9.7609e-2 (1.08e-3) =	<b>9.7049e-2 (9.71e-4) =</b>	9.7143e-2 (1.31e-3)
	LIRCMOP11	8.5078e-3 (1.75e-2) -	1.0368e-2 (2.11e-2) -	1.2654e-2 (2.39e-2) -	5.9571e-3 (1.27e-2) -	<b>2.6556e-3 (1.54e-4)</b>
	LIRCMOP12	4.3056e-3 (9.08e-4) =	3.8896e-3 (4.14e-4) =	4.1214e-3 (5.71e-4) =	4.5156e-3 (1.56e-3) =	<b>3.7282e-3 (3.54e-4)</b>
	LIRCMOP1	3.5377e-2 (3.21e-2) =	4.9291e-2 (5.24e-2) =	4.1292e-2 (3.20e-2) =	3.8397e-2 (3.64e-2) =	<b>2.5642e-2 (8.73e-3)</b>
	LIRCMOP2	1.2925e-2 (1.71e-2) -	1.5042e-2 (1.37e-2) -	1.5808e-2 (1.45e-2) -	4.6502e-3 (2.73e-3) -	<b>4.2606e-3 (1.66e-4)</b>
III	LIRCMOP3	3.9185e-2 (6.30e-2) -	1.7122e-2 (2.82e-2) -	3.6179e-2 (4.66e-2) -	4.2826e-2 (5.41e-2) -	<b>2.6198e-3 (1.65e-3)</b>
	LIRCMOP4	1.4824e-2 (2.19e-2) -	2.0253e-2 (3.38e-2) -	2.8587e-2 (4.19e-2) -	2.0637e-2 (3.32e-2) -	<b>2.3156e-3 (1.81e-3)</b>

LIRCMOP7	7.4896e-3 (2.65e-4) -	7.5199e-3 (2.93e-4) -	7.6988e-3 (4.47e-4) -	7.8608e-3 (9.47e-4) -	7.2760e-3 (1.97e-4)
LIRCMOP8	7.4170e-3 (2.19e-4) -	7.3483e-3 (2.05e-4) -	7.4499e-3 (2.54e-4) -	7.4190e-3 (2.36e-4) -	7.2325e-3 (1.93e-4)
-	+/-/=	1/8/5	2/7/5	2/8/4	2/8/4

Table S13 Adaptive stage transition and the CIDEMT algorithm's HV results on different types of problems

Type	Problem	S- CIDEMT-1	S- CIDEMT-2	S- CIDEMT-3	S- CIDEMT-4	CIDEMT
I	LIRCMOP5	2.9107e-1 (2.51e-4) =	2.9113e-1 (2.68e-4) =	<b>2.9115e-1 (2.16e-4) =</b>	2.9112e-1 (2.23e-4) =	2.9107e-1 (3.20e-4)
	LIRCMOP6	1.9600e-1 (2.01e-4) =	1.9595e-1 (2.14e-4) =	1.9596e-1 (1.67e-4) =	1.9595e-1 (2.24e-4) =	<b>1.9600e-1 (2.25e-4)</b>
	LIRCMOP9	5.5824e-1 (6.57e-3) +	5.5540e-1 (8.22e-3) +	<b>5.5919e-1 (4.61e-3) +</b>	5.5669e-1 (7.91e-3) +	5.4768e-1 (7.49e-3)
	LIRCMOP10	7.0441e-1 (3.66e-4) -	7.0543e-1 (4.66e-4) +	<b>7.0555e-1 (5.23e-4) +</b>	7.0529e-1 (6.59e-4) +	7.0461e-1 (3.69e-4)
	LIRCMOP13	5.3544e-1 (4.44e-3) -	5.3533e-1 (3.54e-3) -	5.3655e-1 (3.65e-3) =	5.3548e-1 (3.57e-3) -	<b>5.3837e-1 (2.65e-3)</b>
II	LIRCMOP14	5.5006e-1 (1.91e-3) =	5.4998e-1 (2.42e-3) =	5.4954e-1 (1.70e-3) -	5.5036e-1 (1.70e-3) =	<b>5.5062e-1 (1.63e-3)</b>
	LIRCMOP11	6.9065e-1 (8.39e-3) -	6.9001e-1 (1.01e-2) =	6.8883e-1 (1.14e-2) -	6.9198e-1 (6.15e-3) -	<b>6.9388e-1 (6.65e-5)</b>
	LIRCMOP12	6.1990e-1 (4.46e-4) -	6.2006e-1 (2.13e-4) =	6.2001e-1 (2.26e-4) -	6.1978e-1 (8.35e-4) -	<b>6.2012e-1 (1.48e-4)</b>
	LIRCMOP1	2.2056e-1 (1.97e-2) =	2.1284e-1 (2.80e-2) =	2.1747e-1 (2.11e-2) =	2.1977e-1 (2.09e-2) =	<b>2.2954e-1 (3.11e-3)</b>
	LIRCMOP2	3.5527e-1 (1.12e-2) -	3.5400e-1 (8.95e-3) -	3.5389e-1 (8.54e-3) -	3.6063e-1 (1.37e-3) -	<b>3.6073e-1 (1.49e-4)</b>
III	LIRCMOP3	1.9268e-1 (2.39e-2) -	2.0138e-1 (1.30e-2) -	1.9329e-1 (2.06e-2) -	1.9027e-1 (2.32e-2) -	<b>2.0818e-1 (1.21e-3)</b>
	LIRCMOP4	3.1110e-1 (1.14e-2) -	3.0813e-1 (1.77e-2) -	3.0526e-1 (1.86e-2) -	3.0865e-1 (1.62e-2) -	<b>3.1766e-1 (8.24e-4)</b>
	LIRCMOP7	2.9435e-1 (2.30e-4) -	2.9423e-1 (3.58e-4) -	2.9416e-1 (4.13e-4) -	2.9406e-1 (7.26e-4) -	<b>2.9447e-1 (9.05e-5)</b>
	LIRCMOP8	2.9441e-1 (1.80e-4) -	2.9441e-1 (1.22e-4) -	2.9439e-1 (1.16e-4) -	2.9439e-1 (1.59e-4) -	<b>2.9452e-1 (8.88e-5)</b>
-	+/-/=	1/9/4	2/7/5	2/8/4	2/8/4	-

### 3. Optimization attempts in high-dimensional problems

To further demonstrate the versatility of the CIDEMT algorithm in solving constrained multi-objective optimization problems, we conducted experiments on the high-dimensional LSCM [2] problem, where all instances were constrained multi-objective problems with a dimensionality of 100. The detailed results are presented in Tables S14 and S15. While CIDEMT outperformed most benchmark algorithms in many cases, the optimization results did not meet expectations. This suggests that there is still room for improvement in CIDEMT's optimization coverage, particularly for complex high-dimensional problems.

Table S14 The IGD results of CIDEMT and the benchmark algorithm in the LSCM problem

Problem	CMOES	EMCMO	CMOEMT	MTCMO	APSEA	IMCMOEA	CIDEMT
LSCM1	2.8440e-1 (2.21e-2) -	2.8480e-1 (1.77e-2) -	3.0267e-1 (2.38e-2) -	2.8518e-1 (1.70e-2) -	2.7991e-1 (2.23e-2) -	3.8507e-1 (4.12e-2) -	<b>2.5086e-1 (2.25e-2)</b>
LSCM2	1.0002e+1 (1.16e+1) -	2.7769e+1 (3.94e+1) -	4.0021e+1 (6.82e+1) -	2.3568e+0 (3.63e+0) -	4.7392e+0 (1.22e+1) -	4.4548e+2 (7.05e+2) -	<b>1.2399e+0 (2.41e+0)</b>
LSCM3	6.3355e-1 (1.58e-1) =	6.6416e-1 (1.31e-1) =	6.7779e-1 (1.96e-1) =	<b>5.3747e-1 (1.11e-1) =</b>	6.6545e-1 (1.54e-1) =	2.3767e+0 (5.28e-1) -	6.1218e-1 (1.47e-1)
LSCM4	1.7840e+1 (1.21e+1) -	3.4933e+1 (3.68e+1) -	9.8877e+1 (1.77e+2) -	3.6027e+0 (8.86e+0) =	5.3080e+0 (8.39e+0) =	3.9681e+2 (6.66e+2) -	<b>3.0764e+0 (1.88e+0)</b>
LSCM5	6.0400e-1 (7.97e-2) +	7.4177e-1 (1.02e-1) =	<b>5.1835e-1 (4.65e-2) +</b>	8.3697e-1 (1.64e-1) -	7.5920e-1 (1.86e-1) =	6.9212e-1 (1.55e-1) =	8.0001e-1 (3.31e-1)
LSCM6	2.1776e-1 (3.70e-2) +	2.9203e-1 (9.60e-2) -	2.5640e-1 (4.61e-2) -	<b>2.0824e-1 (3.74e-2) +</b>	2.5982e-1 (9.98e-2) -	3.6540e-1 (7.74e-2) -	2.3050e-1 (2.20e-2)
LSCM7	5.3225e-1 (1.02e-1) =	6.0285e-1 (8.59e-2) -	7.1826e-1 (2.74e-1) -	<b>3.0529e-1 (7.53e-2) +</b>	8.5302e-1 (2.68e-1) -	8.5601e-1 (2.22e-1) -	5.4913e-1 (1.01e-1)
LSCM8	1.9021e+0 (6.98e+0) =	6.1096e-1 (1.41e-1) =	5.8565e-1 (1.10e-1) =	<b>4.3404e-1 (1.48e-1) +</b>	6.2355e-1 (1.38e-1) =	1.1347e+0 (3.96e-1) -	5.6446e-1 (1.13e-1)
LSCM9	1.2209e+0 (1.32e+0) -	1.2665e+2 (1.44e+2) -	2.1600e+1 (5.47e+1) -	4.5833e+1 (1.26e+2) -	1.3745e+2 (1.67e+2) -	5.9188e+2 (4.56e+2) -	<b>2.1519e-1 (1.21e-1)</b>
LSCM10	6.3222e-1 (2.67e-1) +	6.0820e-1 (2.52e-1) +	8.8727e-1 (2.36e-1) +	<b>4.6392e-1 (1.17e-1) +</b>	6.0302e-1 (2.45e-1) +	5.1679e+0 (4.01e+0) -	1.1580e+0 (4.18e-1)
LSCM11	4.4932e+0 (5.08e+0) -	7.3166e+1 (4.56e+1) -	2.1571e+1 (3.01e+1) -	1.9376e+1 (3.92e+1) -	6.0205e+1 (5.51e+1) -	3.8892e+2 (5.57e+2) -	<b>1.4188e+0 (7.16e-1)</b>
LSCM12	8.8758e+0 (1.20e+1) -	2.6876e+1 (4.00e+1) -	5.3522e+1 (1.59e+2) -	1.5777e+0 (8.07e-1) -	4.9996e+0 (8.25e+0) -	3.6854e+2 (3.10e+2) -	<b>8.4192e-1 (2.89e-1)</b>
+/-/=	3/6/3	1/8/3	2/8/2	4/6/2	1/6/5	0/11/1	

Table S15 The HV results of CIDEMT and the benchmark algorithm in the LSCM problem

Problem	CMOES	EMCMO	CMOEMT	MTCMO	APSEA	IMCMOEA	CIDEMT
LSCM1	2.1479e-1 (3.22e-2) -	2.1290e-1 (2.30e-2) -	2.2468e-1 (3.61e-2) -	2.1638e-1 (2.29e-2) -	2.2078e-1 (3.00e-2) -	6.2547e-2 (4.10e-2) -	<b>2.7650e-1 (1.96e-2)</b>
LSCM2	1.5067e-3 (8.25e-3) -	0.0000e+0 (0.00e+0) -	5.6939e-5 (3.12e-4) -	2.9731e-2 (3.44e-2) -	6.6965e-3 (1.49e-2) -	0.0000e+0 (0.00e+0) -	<b>8.8991e-2 (1.70e-2)</b>
LSCM3	3.0672e-2 (3.51e-2) =	2.6693e-2 (3.46e-2) =	2.1448e-2 (3.29e-2) =	2.7572e-2 (3.34e-2) =	2.6496e-2 (3.45e-2) =	0.0000e+0 (0.00e+0) -	<b>3.6102e-2 (3.99e-2)</b>
LSCM4	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	<b>8.5085e-3 (2.39e-2) =</b>	2.5524e-3 (1.27e-2) =	0.0000e+0 (0.00e+0) =	3.8660e-3 (1.38e-2)
LSCM5	1.0879e-3 (5.96e-3) -	0.0000e+0 (0.00e+0) -	<b>9.4058e-2 (3.81e-2) +</b>	0.0000e+0 (0.00e+0) -	4.8451e-3 (1.64e-2) -	1.8939e-2 (2.62e-2) =	1.5515e-2 (3.35e-2)
LSCM6	2.7167e-1 (5.13e-2) -	2.0846e-1 (9.69e-2) -	2.3673e-1 (5.59e-2) -	2.9811e-1 (4.79e-2) -	2.4479e-1 (9.63e-2) -	1.7200e-1 (5.90e-2) -	<b>3.0917e-1 (2.44e-2)</b>
LSCM7	6.7954e-2 (4.96e-2) =	3.3489e-2 (5.30e-2) -	4.4270e-2 (6.13e-2) =	<b>2.0352e-1 (8.38e-2) +</b>	1.3652e-2 (2.22e-2) -	8.8793e-3 (1.29e-2) -	5.7787e-2 (4.50e-2)
LSCM8	4.4809e-2 (4.10e-2) -	3.7531e-2 (4.01e-2) -	2.8311e-2 (3.12e-2) -	5.7910e-2 (4.18e-2) -	2.7294e-2 (3.48e-2) -	4.3433e-4 (2.38e-3) -	<b>7.2036e-2 (4.26e-2)</b>
LSCM9	2.0604e-2 (3.14e-2) -	0.0000e+0 (0.00e+0) -	2.0704e-2 (2.94e-2) -	1.2723e-2 (2.92e-2) -	0.0000e+0 (0.00e+0) -	0.0000e+0 (0.00e+0) -	<b>1.4116e-1 (5.94e-2)</b>
LSCM10	1.6563e-1 (9.25e-2) +	1.8373e-1 (1.24e-1) +	3.9791e-2 (6.58e-2) =	<b>2.5943e-1 (1.22e-1) +</b>	1.8829e-1 (1.26e-1) +	8.5137e-4 (3.58e-3) -	2.5238e-2 (5.09e-2)
LSCM11	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	0.0000e+0 (0.00e+0) =	<b>1.4775e-2 (4.53e-2) =</b>	2.9639e-4 (1.62e-3) =	0.0000e+0 (0.00e+0) =	4.1449e-3 (1.53e-2)
LSCM12	4.6692e-3 (1.82e-2) -	4.7558e-3 (1.81e-2) -	1.9580e-2 (3.76e-2) -	8.6871e-2 (4.58e-2) -	3.5600e-2 (4.65e-2) -	0.0000e+0 (0.00e+0) -	<b>1.6967e-1 (2.13e-2)</b>
+/-/=	1/7/4	1/8/3	1/6/5	2/5/5	1/8/3	0/9/3	

### References

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