Lumped-parameter modeling of the cardiovascular system and ECMO pump

In the lumped parameter model of the cardiovascular system, each vascular segment is represented by a three-element Windkessel unit consisting of a resistance (R), a compliance (C), and an inertance (L). Details of the vascular network and assigned model parameter values have been reported in our previous work [1]. The governing equations for blood flow are formulated following the mass and momentum conservation principles, which are defined at the pressure (‘P’) and flow (‘Q’) nodes, respectively.

At a ‘P’ node, the equation describing mass conservation reads

 (S1)

At a ‘Q’ node, the momentum conservation equation is expressed as

 (S2)

where *t* is the time, subscripts ‘*j* − 1’, ‘*j*’, and ‘*j* + 1’ denote the upstream, current, and downstream vascular compartments, respectively.

The model also contains lumped parameter representations of the heart and cardiac valves. Detailed descriptions of the modeling approach can be found in our earlier publications [2,3]. In brief, the contraction-relaxation behavior of each cardiac chamber is mathematically described by a time-dependent elastance function. Each cardiac valve is modeled as a one-way valve incorporated with viscous and Bernoulli’s resistances, which can account for trans-valvular pressure drop while ensuring unidirectional blood flow.

Similar to the cardiovascular system, the VA-ECMO system is modeled by representing the blood pump, inflow cannula, and outflow cannula with lumped parameters. In the sub-model of the blood pump, the pump flow rate () is expressed as a function of the rotational speed () and the pressure gradient across the pump () [4].

(S3)

where *K*A was -0.0018 , *K*B was -1.2 \* 10-5 , and *K*C was 7.3 × 10-6 .

References

1. D. Li, X. Li, Y. Xia, C. Weng, F. Liang, “Impact of peripheral venoarterial extracorporeal membrane oxygenation support for heart failure on systemic hemodynamics and aortic blood flow, ” Physics of Fluids, 36 (10): 101916 (2024).
2. F. Liang, H. Senzaki, C. Kurishima, K. Sughimoto, R. Inuzuka, and H. Liu, “Hemodynamic performance of the Fontan circulation compared with a normal biventricular circulation: a computational model study,” American Journal of Physiology-Heart and Circulatory Physiology 307, H1056–H1072 (2014).
3. W. Cui, T. Wang, Z. Xu, J. Liu, S. Simakov, and F. Liang, “A numerical study of the hemodynamic behavior and gas transport in cardiovascular systems with severe cardiac or cardiopulmonary failure supported by venoarterial extracorporeal membrane oxygenation,” Frontiers in Bioengineering and Biotechnology 11, 1177325 (2023).
4. Y. Shi and T. Korakianitis, “Impeller-pump model derived from conservation laws applied to the simulation of the cardiovascular system coupled to heart-assist pumps,” Computers in Biology and Medicine 93, 127–138 (2018).