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Article

Optimal Control and Spectral Collocation Method for Solving Smoking Models

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Abstract: In this manuscript, we solve the ordinary model of nonlinear smoking mathematically by using the second kind of shifted Chebyshev polynomials. The stability of the equilibrium point is calculated. The schematic of the model illustrates our proposition. We discuss the optimal control of this model, and formularize the optimal control smoking work through the necessary optimality cases. A numerical technique for the simulation of the control problem is adopted. Moreover, a numerical method is presented, and its stability analysis discussed. Numerical simulation then demonstrates our idea. Optimal control for the model is further discussed by clarifying the optimal control through drawing before and after control. Fractional request differential equations (FDEs) are usually used to display frameworks that have memory and exist in a few thermoelasticity models and organic standards. FDEs show the realistic biphasic decline of infection of diseases but at a slower rate. FDEs are more suitable than integer order ones in modeling complex systems, such as biological systems.

Keywords: Shifted second kind chebyshev; smoking model; stability; hamiltonian; lagrange multipliers; optimal control

1 Introduction

Infectious diseases have a tremendous influence on human life. Every year billions of people suffer from or die due to various infectious diseases. Mathematical modeling is of considerable importance in epidemiology because it may provide understanding of the underlying mechanisms that influence the spread of a disease and thus may be used to offer control strategies. Many scientists explored the detection of illnesses and pests [1–72], wherein the idea that mathematical modeling helps in discovering diseases' spread [17]. In 1766, Islamic scholars presented for the first time what was considered the beginning of modern epidemiology discovery to be since followed. Model smoking is one of the most famous and important model commonly used by researchers [8–14], [18–21], [22–36]. According to the World Health Organization [14], the global tobacco pandemic killed several million people. The cost of health care is increasing and its budgets are decreasing as developing economies are shrinking [17]. Smoking negatively affects overall health as well [1–4]. According to [1–3], more than five million individuals are killed every year from smoking tobacco, as such it is the main cause of preventable deaths. Serious illnesses caused by it include asthma, lung cancer, heart disease, and mouth ulcers. In

[8–9], we discussed the smoking dynamics of the integer {of the integer what}. Moreover, [13] analyzed the above using the Homotopy analysis method. Four kinds of polynomials, Chebyshev [35], [57–59]: Chebyshev's model includes four kinds of polynomials. The outcomes of each and their applications can be found in several books. Moreover, many researchers use several of these polynomials in a single model [33,35]. However, the shifted Chebyshev polynomial of the second kind $Z_n^*(y)$ still needs further study [35]. The model in [56] has proven the equilibrium points.

In the present manuscript, we discuss the analytical solutions for alpha = 1, for the following order of smoking models [4,5,56]:

$$\begin{aligned} DP &= a(1-P) - bPS, \\ DL &= -aL + bPL - cLS, \\ DS &= -(a+d)S + cLS + fQ, \\ DQ &= -(a+f)Q + d(1-e)S, \\ DR &= -aR + edS. \end{aligned} \tag{1}$$

The initial conditions are as follows:

$$\begin{aligned} &P_0 = 0.55, \quad L_0 = 0.2, \quad S_0 = 0.17272, \quad R_0 = 0.01028, \quad f = 0.25, \quad e = 0.4, \quad d = 0.2, \quad c = 0.3, \\ &\quad a = 0.04, \quad b = 0.23, \quad D = \frac{d}{dt}. \end{aligned} \tag{2}$$

In Fig. 1, a schematic proposition graph is a tool that is applied to display the interrelation between model states and enables the application of graph-theoretic tools to discover novel features of the model.

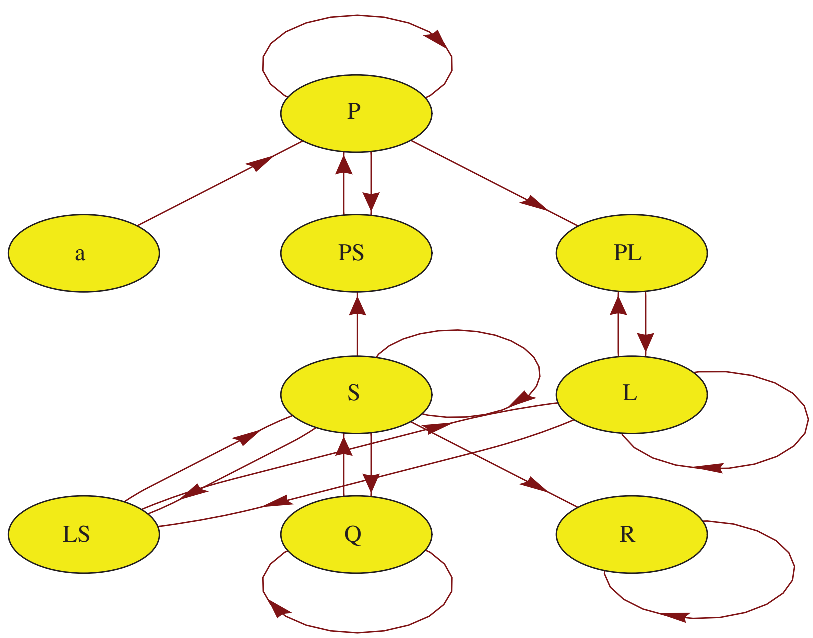

Figure 1: Schematic of the proposed model

Tab. 1 describes the parameters of the model.

The organization of the remainder of this paper is as follows. The stability of equilibrium points is studied in Section 2. Optimal control for the smoking model is discussed in Section 3. In Section 4, the second kind of Chebyshev polynomials and their properties are given. In Section 5, the numerical implementation is given. Finally, a conclusion is presented in Section 6.

Table 1: Parameter definitions

T(t)	Total population with time
P(t)	Probable smokers
L(t)	Occasional smokers
S(t)	Heavy smokers
Q(t)	Temporary quitters
R(t)	Smokers who quit permanently
a	Rate of natural death
b	Contact rate between probable and occasional smokers
c	Contact rate between occasional and smokers heavy
d	Average number of [x] who quit smoking
e	The fraction of remaining smokers
f	Contact rate between smokers and temporary quitters

2 Stability of Equilibrium Points

For the stability behavior of this model at E0=(1, 0, 0, 0, 0), we use transformations [36]:

U = 1-P, P = 1-U, L=L, S=S, Q=Q, R=R,

U*= –P*, L*=L*, S*=S*, Q*=Q*, R*=R*.

It follows that

$$\begin{aligned} \frac{dU}{dt} &= -aU + bS - bSU, \\ \frac{dL}{dt} &= -aL + bL - bLU - cLS, \\ \frac{dS}{dt} &= -aS - dS + cLS + fQ, \\ \frac{dQ}{dt} &= -aQ - fQ + ds - deS, \\ \frac{dR}{dt} &= -aR + edS. \end{aligned} \tag{2.1}$$

The system (2.1) has

P1=(0, 0, 0, 0, 0), P2=(U*, L*, S*, Q*, R*), P3=(U**, L**, S**, Q**, R**),

P4=(U***, L***, S***, Q***, R***) and P5=(U****, L****, S****, Q****, R****),

where, U*=1-P*, U**=1-P**, U***=1-P*** and U****=1-P****.

Through Taylor approximation, the linearized form of the model is

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ V' \\ W' \end{bmatrix} = B \begin{bmatrix} X \\ Y \\ Z \\ V \\ W \end{bmatrix}, \quad (2.2)$$

where X=U-0, Y=L-0, Z=S-0, V=Q-0, W=R-0, and

$$\mathrm{B} = \begin{bmatrix} -\mathrm{a} & 0 & \mathrm{b} & 0 & 0 \\ 0 & -\mathrm{a}+\mathrm{b} & 0 & 0 & 0 \\ 0 & -\mathrm{a}-\mathrm{d} & \mathrm{f} & 0 & 0 \\ 0 & 0 & \mathrm{d}-\mathrm{de} & -\mathrm{a}-\mathrm{f} & 0 \\ 0 & 0 & \mathrm{ed} & 0 & -\mathrm{a} \end{bmatrix}. \quad (2.3)$$

The stability of (2.1) is the same as the linearized stability (2.3). The stability of (2.3) depends on eigenvalues *B*. The point *P1* is asymptotically locally stable if eigenvalues are real negative parts; conversely, *P1* is unstable if at least one eigenvalue of *B* is a nonnegative real part. For details on stability see [56,63].

3 Optimal Control for Smoking Model

Consider the state presented (1.2), in R^5, with control functions admissible [28–32]:

$$\Omega = \{(u_V(.),\ u_R(.)) \in (L^{\infty}(0,\ T_f)^2) |\ 0 \le u_V(.),\ u_R(.) \le 1, \quad \forall t \in [0,\ T_f]\},$$

where T_f is the final time, and $u_V(.)$ and $u_R(.)$ are functions controls.

The objective function is defined as

$$J(u_V(.),\ u_R(.)) = \int_0^{T_f} [AL(t) + Bu_V^2(t) + Cu_R^2(t)]dt, \quad (3.1)$$

where A, B, and C represent the number of occasional smokers, the rate of contact between smokers and quitters, and temporarily who regain support to smoking, and rate of smoking quitting.

We minimize the objective function as follows [28–32]:

$$J(u_V,\ u_R) = \int_0^{T_f} \eta\, dt, \quad (3.2)$$

which is subjected to the constraint

$$DP = \xi_1, \quad DL = \xi_2, \ DS = \xi_3, \ DQ = \xi_4, \quad DR = \xi_5, \quad \xi_k = \xi, \ k = 1,\ 2,\ 3,\ 4,\ 5. \quad (3.3)$$

The following initial conditions are satisfied:

$$P = P_0, \quad L = L_0, \quad S = S_0, \quad Q = Q_0, \quad R = R_0. \quad (3.4)$$

OCP is defined, and we consider the following modified objective (cost) function:

$$\overline{J} = \int_0^{T_f} \left[H - \sum_{k=1}^{5} \lambda_k \xi_k \right] dt, \tag{3.5}$$

where the Hamiltonian and control smoking objective functions are defined as

$$H = \eta + \sum_{k=1}^{5} \lambda_k \xi_k, \tag{3.6}$$

$$\begin{aligned} H = {} & AL + Bu_E^2 + Cu_M^2 + \lambda_1^*(a(1-P) - bPS) + \lambda_2^*(-aL + bPL - cLS) \\ & + \lambda_3^*(-(a+d)S + cLS + fQ) + \lambda_4^*(-(a+f)Q + d(1-e)S) + \lambda_5^*(-aR + edS). \end{aligned} \tag{3.7}$$

From (3.5) and (3.7), the conditions, necessary and sufficient for OPC are

$$D\lambda_1^* = \frac{\partial H}{\partial P}, \quad D\lambda_2^* = \frac{\partial H}{\partial L}, \quad D\lambda_3^* = \frac{\partial H}{\partial S}, \quad D\lambda_4^* = \frac{\partial H}{\partial Q}, \quad D\lambda_5^* = \frac{\partial H}{\partial R}, \tag{3.8}$$

$$0 = \frac{\partial H}{\partial u_k} \Rightarrow \quad 0 = \frac{\partial H}{\partial u_E}, \quad 0 = \frac{\partial H}{\partial u_M}. \tag{3.9}$$

$$DP = \frac{\partial H}{\partial \lambda_1^*}, \quad DL = \frac{\partial H}{\partial \lambda_2^*}, \quad DS = \frac{\partial H}{\partial \lambda_3^*}, \quad DQ = \frac{\partial H}{\partial \lambda_4^*}, \quad DR = \frac{\partial H}{\partial \lambda_5^*}. \tag{3.10}$$

$$\lambda_k, \ (T_f) = 0. \tag{3.11}$$

where λ_k, k = 1, 2, 3, 4, 5 are Lagrange multipliers. Eqs. (3.9)–(3.10) clarify the conditions of the Hamiltonian for the OPC.

We construct a theorem similar to that presented in [28–32], [44–47].

Theorem 1.

If u_V and u_R are optimal controls with states corresponding to P^*, L^*, S^*, Q^* and R^*, there work out adjoint variables λ_k^*, $k = 1, 2, 3, 4, 5$, accepts:

i) Co-state equation

$$D\lambda_1^* = \lambda_1^*(-a - bS^*) + \lambda_2^*(bL^*), \tag{3.12}$$
$$D\lambda_2^* = A + \lambda_2^*(-a + bP^*) + \lambda_3^*(cS^*), \tag{3.13}$$
$$D\lambda_3^* = \lambda_1^*(-bP^*) + \lambda_2^*(-cL^*) + \lambda_3^*(-a - d + cL^*) + \lambda_4^*(d - ed) + \lambda_5^*(ed), \tag{3.14}$$
$$D\lambda_4^* = \lambda_3^*(f) + \lambda_4^*(-a - f), \tag{3.15}$$
$$D\lambda_5^* = \lambda_5^*(-a). \tag{3.16}$$

ii) With condition transversality:

$$\lambda_k^*(T_f) = 0. \tag{3.17}$$

iii) Optimality conditions

$$H = \min_{0 \le u_V^*, u_R^* \le 1} H, \tag{3.18}$$

Therefore, the function controls u_V^*, u_R^* are given by

$$u_V^* = \frac{Q^*[\lambda_4^* - \lambda_3^*]}{2B}, \tag{3.19}$$

$$u_R^* = \frac{S^*[\lambda_3^* - \lambda_4^* - e\lambda_5^*]}{2C}, \tag{3.20}$$

$$u_V^* = \min\left\{1, \quad \max\left\{0, \frac{Q^*[\lambda_4^* - \lambda_3^*]}{2B}\right\}\right\}, \tag{3.21}$$

$$u_R^* = \min\left\{1, \quad \max\left\{0, \frac{S^*[\lambda_3^* - \lambda_4^* - e\lambda_5^*]}{2C}\right\}\right\}. \tag{3.22}$$

For more on optimal controls for solving models, see [29–32,44,47].

4 Properties of the Second Kind of Chebyshev Polynomials

4.1 Second Kind of Chebyshev Polynomials

Chebyshev polynomials $Z_n(y)$ of the second kind are rectangular polynomials of stage n in x presented on the interval [−1, 1] [34,35,57–59]:

$Z_n(y) = \frac{\sin(n+1)\theta}{\sin\theta}$, where $y = \cos\theta$ and $\theta \in [0, \pi]$.

Polynomials have rectangular with rating to the products indoor

$$\langle Z_n(y),\ Z_m(y)\rangle = \int_{-1}^{1} \sqrt{1-y^2} Z_n(y) Z_m(y) dy = \begin{cases} 0, & n \neq m, \\ \dfrac{\pi}{2}, & n = m, \end{cases} \tag{4.1}$$

where $\sqrt{1-y^2}$ is a weight function.

$Z_n(y)$ can be generated by recurring relations

$Z_n(y) = 2y\, Z_{n-1}(y) - Z_{n-2}(y)$, n = 2, 3, …, n, with $Z_0(y) = 1$. $Z_1(y) = 2y$.

The analytical form $Z_n(y)$ of stage n is given by [34]:

$$Z_n(y) = \sum_{i=0}^{\lceil \frac{n}{2} \rceil} (-1)^i \binom{n-i}{i} (2y)^{n-2i} = \sum_{i=0}^{\lceil \frac{\pi}{2} \rceil} (-1)^i 2^{n-2i} \frac{\Gamma(n-i+1) y^{n-2i}}{\Gamma(i+1)\Gamma(n-2i+1)}, \quad n > 0. \tag{4.2}$$

Here, $\left\lceil \frac{n}{2} \right\rceil$ is part integral n/2.

4.2 Second Kind of Shifted Chebyshev Polynomials

On the interval $y \in [0, 1]$, $Z_n^*(y)$is defined via substituting a change variable $z = 2y - 1$. Hence, $Z_n^*(y)$is defined as [33–35] $Z_n^*(y) = Z_n(2y-1)$, and satisfies the following relation: $2yZ_{n-1}^*(y^2) = Z_{2n-1}(y)$.

The following inner product is orthogonal on the interval [0, 1]:

$$\langle Z_n^*(y),\ Z_m^*(y)\rangle = \int_{0}^{1} \sqrt{y-y^2} Z_n^*(y) Z_m^*(y) dy = \begin{cases} 0, & n \neq m, \\ \dfrac{\pi}{8}, & n = m, \end{cases} \tag{4.3}$$

with weight function $\sqrt{y-y^2}$.$Z_n^*(y)$ can be produced via recurring relations

$Z_n^*(y) = 2(2y-1)Z_{n-1}^*(y) - Z_{n-2}^*(y)$, n = 2, 3, …, n, with $Z_0^*(y) = 1$, $Z_0^*(y) = 4y - 2$.

The following formula represents the analytical

$$Z_r^*(y) = \sum_{j=0}^{r} (-1)^j 2^{2r-2j} \frac{\Gamma(2r-j+2)y^{r-j}}{\Gamma(j+1)\Gamma(2r-2j+2)}, \quad r > 0, \tag{4.4}$$

The solution of this model can be written as $Z^*(y)$.

Let g(y) be a square integral in [0, 1], and then the second kind of shifted Chebyshev polynomials can be represented as follows:

$$g(y) = \sum_{i=0}^{\infty} a_i Z_i^*(y), \tag{4.5}$$

The coefficients a_j, j = 0, 1, …, are expressed as

$$a_i = \frac{2}{\pi}\int_{-1}^{1} g(\frac{y+1}{2})\sqrt{1-y^2} Z_i(y)dy, \tag{4.6}$$

or

$$a_i = \frac{8}{\pi}\int_{0}^{1} g(y)\sqrt{y-y^2} Z_i^*(y)dy, \tag{4.6a}$$

We use only the (r + 1) terms. Then,

$$g_r(y) = \sum_{i=0}^{r} a_i Z_i^*(y). \tag{4.7}$$

Using the practice to construct an integral collocation style then z(y)as follows [33]:

$$\frac{d^k z(y)}{dx^k} \cong \sum_{n=0}^{r} a_n Z_n^*(y) = \sum_{n=0}^{r} a_n w_n^k(y). \tag{4.8}$$

By integrating (4.8), we can obtain the following:

$$\frac{d^{k-1} z(y)}{dy^{n-1}} \cong \sum_{n=0}^{r} a_n w_n^{k-1}(y) + c_1 \tag{4.9}$$

$$\frac{d^{k-2} z(y)}{dy^{n-2}} \cong \sum_{n=0}^{r} a_n w_n^{(k-2)}(y) + c_1 y + c_2, \ \ldots \tag{4.10}$$

$$\frac{dz(y)}{dx} \cong \sum_{n=0}^{r} a_n w_n^{(1)}(y) + c_1 \frac{y^{k-2}}{(k-2)!} + c_2 \frac{y^{k-3}}{(k-3)!} + \ldots + c_{k-2} y + c_{k-1}, \tag{4.11}$$

$$z(y) = \sum_{n=0}^{r} a_n w_n^{(0)}(y) + c_1 \frac{y^{k-1}}{(k-1)!} + c_2 \frac{y^{k-2}}{(k-2)!} + \ldots + c_{k-1} y + c_k, \tag{4.12}$$

From (4.8) and (4.12), we then have

$$\begin{aligned}
w_n^{(k)}(y) &= \sum_{i=0}^{n}(-1)^i 2^{2n-2i}\frac{\Gamma(2n-i+2)y^{n-i}}{\Gamma(i+1)\Gamma(2n-2i+2)},\\
w_n^{(k-1)}(y) &= \int w_n^{(k)}(y)dy = \sum_{i=0}^{n}(-1)^i 2^{2n-2i}\frac{\Gamma(2n-i+2)y^{n-i+1}}{\Gamma(i+1)\Gamma(2n-2i+2)(n-i+1)},\\
w_n^{(k-2)}(y) &= \int w_n^{(k-1)}(y)dx = \sum_{i=0}^{n}(-1)^i 2^{2n-2i}\frac{\Gamma(2n-i+2)y^{n-i+2}}{\Gamma(i+1)\Gamma(2n-2i+2)(n-i+1)(n-i+2)},\\
w_n^{(0)}(y) &= \int w_n^{(1)}(y)dx = \sum_{i=0}^{n}(-1)^i 2^{2n-2i}\frac{\Gamma(2n-i+2)y^{i+k}}{\Gamma(i+1)\Gamma(2n-2i+2)\ldots(i+k-1)(i+k).}
\end{aligned}\tag{4.13}$$

We presently register Eqs. (4.9)–(4.13) at $(r+1)$ points yp, $p = 0, 1, \ldots, r$ as follows:

$$\begin{aligned}
&\frac{d^k z(y_p)}{dy^k} = \Omega^{(k)}\hat{E}, \quad \frac{d^{k-1}z(y_p)}{dy^{k-1}} = \Omega^{(k-1)}\hat{E}, \ \ldots,\\
&\frac{dz(y_p)}{dy} = \Omega^{(1)}\hat{E}, \quad z(y_p) = \Omega^{(0)}\hat{E},
\end{aligned}\tag{4.14}$$

where $\hat{E} = [a_0, a_1, \ldots, a_m, c_1, c_2, \ldots c_n]^T$, and $\Omega^{(r)}, \Omega^{(r-1)}, \ldots, \Omega^{(0)}$ are combined matrices.

5 Style Integral Collocation for Resolving the Nonlinear Smoking Model

In this section, we present the model's implementation through the following steps:

i) We first approximate the function using Eqs. (4.8)–(4.14) with $m = 5$, as follows:

$$\begin{aligned}
&\frac{dP}{dt} \cong \sum_{n=0}^{5} A_n w_n^{(1)}(t), \quad \frac{dL}{dt} \cong \sum_{n=0}^{5} B_n w_n^{(1)}(t), \quad \frac{dS}{dt} \cong \sum_{n=0}^{5} C_n w_n^{(1)}(t), \quad \frac{dQ}{dt} \cong \sum_{n=0}^{5} D_n w_n^{(1)}(t),\\
&\frac{dR}{dt} \cong \sum_{n=0}^{5} E_n w_n^{(1)}(t), \quad P(t) \cong \sum_{n=0}^{5} A_n w_n^{(0)}(t) + c_1, \quad L(t) \cong \sum_{n=0}^{5} B_n w_n^{(0)}(t) + c_2,\\
&S(t) \cong \sum_{n=0}^{5} C_n w_n^{(0)}(t) + c_3, \quad Q(t) \cong \sum_{n=0}^{5} Q_n w_n^{(0)}(t) + c_4, \quad R(t) \cong \sum_{n=0}^{5} E_n w_n^{(0)}(t) + c_5,
\end{aligned}\tag{5.1}$$

where $w_n^{(0)}(t)$ and $w_n^{(1)}(t)$ are expressed as

$$\begin{aligned}
w_n^{(0)}(t) &= \sum_{i=0}^{n}(-1)^i 2^{2n-2i}\frac{\Gamma(2n-i+2)t^{i+k}}{\Gamma(i+1)\Gamma(2n-2i+2)\ldots(i+k-1)(i+k)},\\
w_n^{(1)}(t) &= \sum_{i=0}^{n}(-1)^i 2^{2n-2i}\frac{\Gamma(2n-i+2)t^{n-i}}{\Gamma(i+1)\Gamma(2n-2i+2)}.
\end{aligned}$$

Then, the nonlinear smoking model (1.1) is transformed to

$$\begin{aligned}
\sum_{n=0}^{5} A_n w_n^{(1)}(t) &= a\left(1-\sum_{n=0}^{5} A_n w_n^{(0)}(t)+c_1\right)-b\left(\sum_{n=0}^{5} A_n w_n^{(0)}(t)+c_1\right)\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t)+c_2\right),\\
\sum_{n=0}^{5} B_n w_n^{(1)}(t) &= -a\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t)+c_2\right)+b\left(\sum_{n=0}^{5} A_n w_n^{(0)}(t)+c_1\right)\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t)+c_2\right)\\
&\quad -c\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t)+c_2\right)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t)+c_3\right),\\
\sum_{n=0}^{5} C_n w_n^{(1)}(t) &= -(a+d)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t)+c_3\right)+c\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t)+c_2\right)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t)+c_3\right)\\
&\quad +f\left(\sum_{n=0}^{5} D_n w_n^{(0)}(t)+c_4\right),\\
\sum_{n=0}^{5} D_n w_n^{(1)}(t) &= -(a+f)\left(\sum_{n=0}^{5} D_n w_n^{(0)}(t)+c_4\right)+(d-de)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t)+c_3\right),\\
\sum_{n=0}^{5} E_n w_n^{(1)}(t) &= -a\left(\sum_{n=0}^{5} E_n w_n^{(0)}(t)+c_5\right)+ed\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t)+c_3\right).
\end{aligned} \tag{5.2}$$

Now, we collocate Eq. (5.2) at $(r+1=6)$ points t_y, $y=0-5$; as follows:

$$\begin{aligned}
\sum_{n=0}^{5} A_n w_n^{(1)}(t_y) &= a\left(1-\sum_{n=0}^{5} A_n w_n^{(0)}(t_y)+c_1\right)-b\left(\sum_{n=0}^{5} A_n w_n^{(0)}(t_y)+c_1\right)\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t_y)+c_2\right),\\
\sum_{n=0}^{5} B_n w_n^{(1)}(t_y) &= -a\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t_y)+c_2\right)+b\left(\sum_{n=0}^{5} A_n w_n^{(0)}(t_y)+c_1\right)\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t_y)+c_2\right)\\
&\quad -c\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t_y)+c_2\right)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t_y)+c_3\right),\\
\sum_{n=0}^{5} C_n w_n^{(1)}(t_p) &= -(a+d)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t_p)+c_3\right)+c\left(\sum_{n=0}^{5} B_n w_n^{(0)}(t_p)+c_2\right)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t_p)+c_3\right)\\
&\quad +f\left(\sum_{n=0}^{5} D_n w_n^{(0)}(t_p)+c_4\right),\\
\sum_{n=0}^{5} D_n w_n^{(1)}(t_p) &= -(a+f)\left(\sum_{n=0}^{5} D_n w_n^{(0)}(t_p)+c_4\right)+(d-de)\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t_p)+c_3\right),\\
\sum_{n=0}^{5} E_n w_n^{(1)}(t_p) &= -a\left(\sum_{n=0}^{5} E_n w_n^{(0)}(t_p)+c_5\right)+ed\left(\sum_{n=0}^{5} C_n w_n^{(0)}(t_p)+c_3\right).
\end{aligned} \tag{5.3}$$

The roots of shifted Chebyshev polynomial $Z_6^*(t)$.

(ii) By putting the initial Eqs. (1.2) into (5.1), we can obtain five equations.

Eqs. (5.1) and (5.3) obtained in step (ii) represent nonlinear algebraic system equations.

(iii) We use Newton's iteration to resolve the system and solve for the unknowns.

Figs. 2–4 show the nonlinear smoking model's behavior before control.

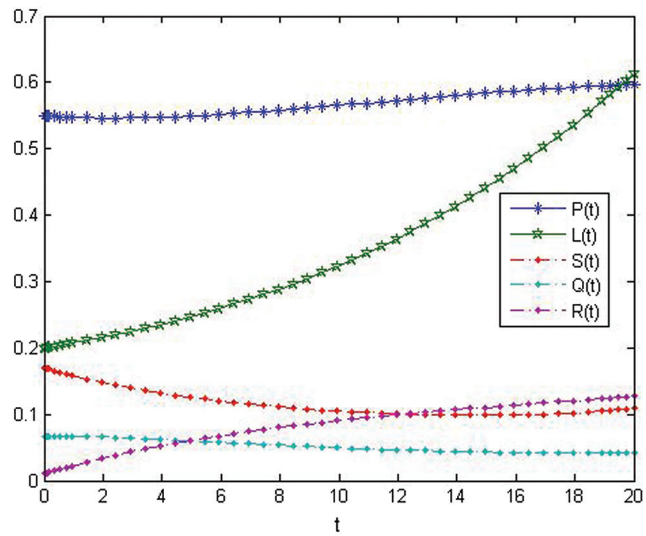

Figure 2: The approximate solution of variables about (ICSM) at r = 5 before control

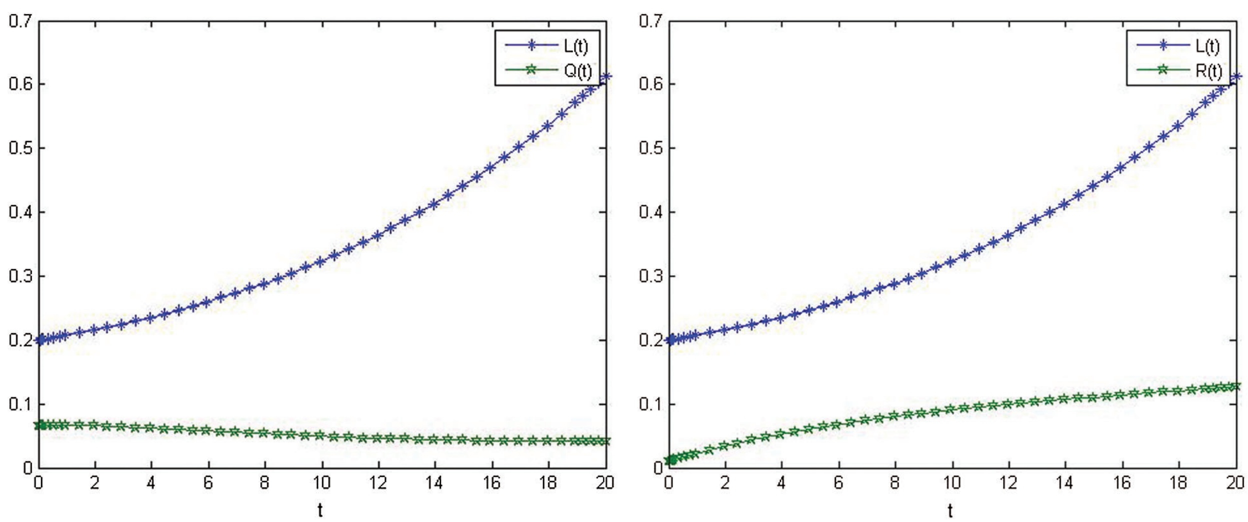

Figure 3: The relationship between L(t) and Q(t) before control. The relationship between L(t) and R(t) before control

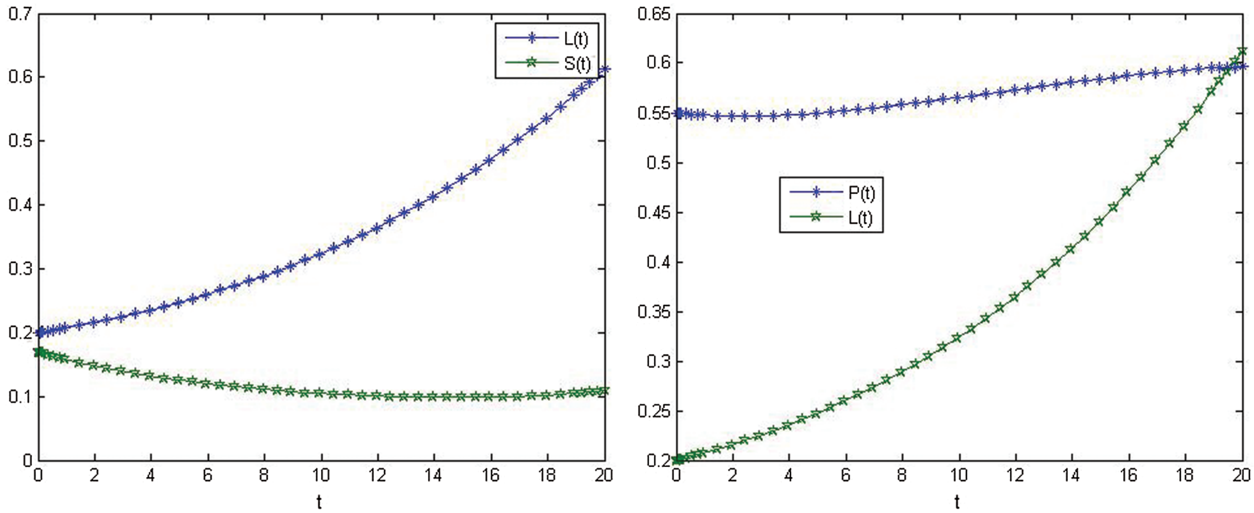

Figure 4: Relationship between L(t) and S(t) before control. Relationship between P(t) and L(t) before control

Figs. 5–8, show the show the nonlinear smoking model's behavior after control.

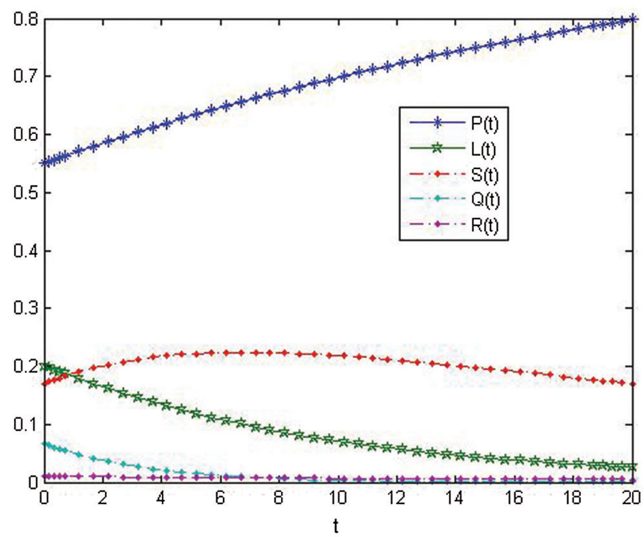

Figure 5: Approximate solution of variables about (ICSM) at r = 5 after control

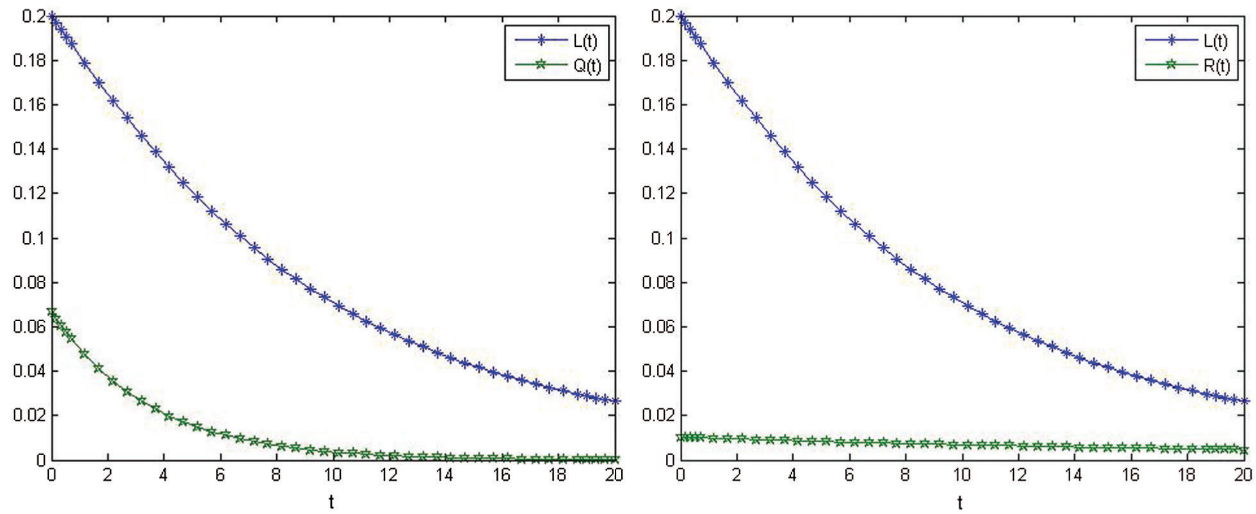

Figure 6: Relationship between L(t) and Q(t) after control. Relationship between L(t) and R(t) after control

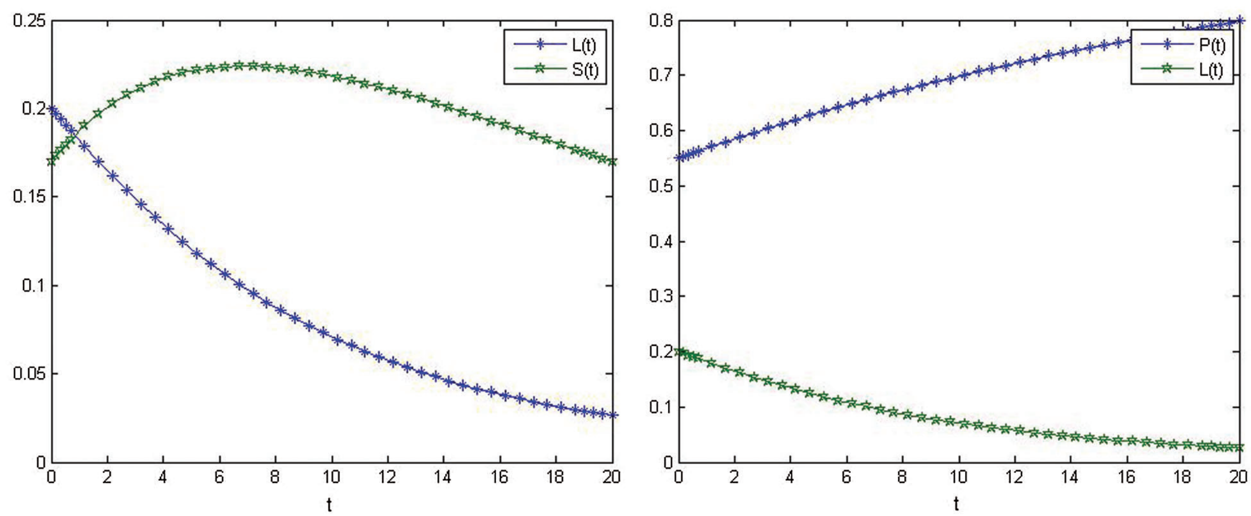

Figure 7: Relationship on L(t) and S(t) after control. Relationship on P(t) and L(t) after control

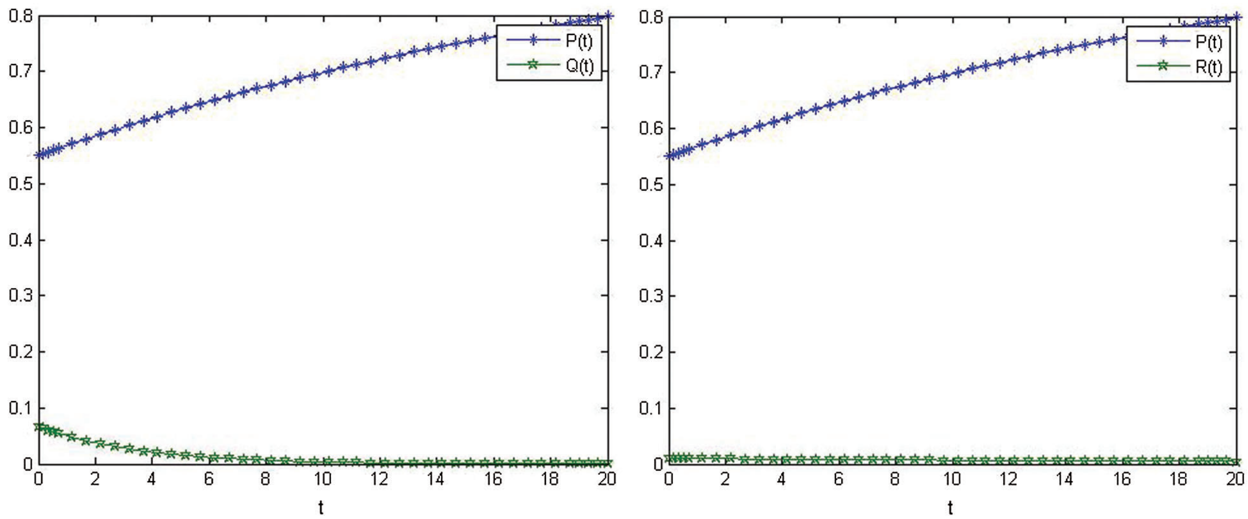

Figure 8: Relationship between P(t) and Q(t) after control. Relationship between P(t) and R(t) after control

6 Conclusions

In this manuscript, a mathematical nonlinear smoking model is studied. The optimal control of this smoking model is discussed. The stability of the equilibrium point is calculated, and a schematic of the proposed model is presented. Moreover, the integral collocation style is used to obtain the approximate solutions of the model. ICM using the shifted Chebyshev polynomials of the second kind is a new technique for solving these problems. Under the application with the necessary optimality conditions, we have studied the problem control with numerical techniques for the simulation. Moreover, a numerical method and its stability is discussed.

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